ELECTROPHORESIS BY ALTERNATING FIELDS IN A NON-NEWTONIAN FLUID

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The possibility of a drift of charged particles in a non-newtonian fluid by a spatially homogeneous time-periodic and non-sine-shaped electric field is proved theoretically. The phenomenon may be important in the transfer accompanied interaction between living cells and dispersive particles and also between intracellular organelles.

1. Electrophoresis [1] and its diversified modifications have been investigated in direct and alternative, homogeneous and nonhomogeneous, fields [2,3]. In this connection, a particle directed drift by an alternative spatially homogeneous field without a direct component is regarded to be impossible. This point of view is true, apparently, when a fluid in which a motion takes place may be described by Newton's model if the time dependence of the field strength has a sine-shaped character. However, there are numerous fluids which are not subject to Newton's model [4]. Distilled water, for example, is traditionally studied in the frame of the Newton model, but under certain conditions it is characterized by a shearing strength [5]. This fact suggests that under this conditions a newtonian description is inadequate. Biofluids, such as intracellular and intercellular fluids, blood, etc., are substantially non-newtonian. It is essential for the forthcoming consideration that a friction force, when a body is moving in a non-newtonian fluid, is nonlinearly velocity-dependent.

On the other hand, time-periodic fields occurring in nature do not necessarily alter according to a simple harmonic law. Multivarious periodic electric signals can be produced by living cells and its various constituents [6]. In particular these can be saw-shaped pulses or the aggregate of several harmonics, and they will be considered below.

Here we consider charged-particle motion in a non-newtonian fluid caused by a periodic electric field. The possibility for directed drift caused by a spatially homogeneous periodic field without a direct component is established. The absence of the direct component means that the electric force applied to a particle equals zero averaged in time. Nevertheless the complete force, composed of electric and friction forces, turns out on the average to be different from zero, due to the nonlinearity of the velocity dependence of the friction force. Caused by nonlinear friction, the directed drift under the action of the electric force (on the average equal to zero) will be named below as nonlinear electrofrictiophoresis (NEFP).

2. We study particle movement neglecting its brownian motion and orientation. In this case the equation of particle motion can be written in terms of the scalar dimensionless velocity

$$\dot{v} + \lambda v - \epsilon G(v) = f(t)\,, \qquad (1)$$

where v is the particle velocity; $\lambda, \epsilon \geqslant 0$; $\epsilon G(v)$ is the nonlinear friction component caused by the deviation from Newton's model; $f(t)$ is a periodic function of unit period which characterizes the force from the field applied to a particle. It follows from physical reasoning that the complete friction force is characterized by a nondecreasing function of velocity:

$$(\mathrm{d}/\mathrm{d}v)\,[\lambda v - \epsilon G(v)] \geqslant 0\,. \qquad (2)$$

The behavior of the solutions to eq. (1) at large times is characterized by the following statement.

Let the condition (2) be fulfilled and the function $f(t)$ be differentiable. Then eq. (1) has a unique solution for all $t \in R$. This solution tends to the unique periodic solution $v^*(t)$ of unit period and is independent of the initial velocity.

When one searches for the solution to eq. (1), the iteration scheme which gives as output of each step the periodic function $v_n^*(t)$, this being the terminal solution to the linear equation, may be used:

$$\dot{v}_0 + \lambda v_0 = f(t), \tag{3}$$

$$\dot{v}_n + \lambda v_n - \epsilon G(v_{n-1}^*) = f(t), \quad n = 1, 2, \dots . \tag{4}$$

The periodic solution to eq. (3) may be obtained according to the formula [7]

$$v_0^*(t) = \frac{1}{e^\lambda - 1} \int_t^{t+1} e^{-\lambda(t-\theta)} f(\theta)\, d\theta .$$

Analogously, the periodic solutions to eq. (4) may be obtained by the formulae

$$v_n^*(t) = \frac{\epsilon}{e^\lambda - 1} \int_t^{t+1} e^{-\lambda(t-\theta)}\, G(v_{n-1}^*(\theta))\, d\theta + v_0^*(t),$$

$$n = 1, 2, \dots .$$

Let us assume that the function $G(v)$ satisfies the following condition

$$\sup |G(v_1) - G(v_2)| \leqslant A |v_1 - v_2|,$$

$$|v_1| \leqslant M, \quad |v_2| \leqslant M,$$

where $M = \sup_{0 \leqslant t < 1} |v^*(t)|$. Then the successive iterations $v_n^*(t)$ converge to the periodic solution to eq. (1) uniformly in t when $n \to \infty$ if $A\epsilon/\lambda < 1$.

3. Let $\langle g \rangle$ denote the time averaged value of the periodic function $g(t)$ with unit period: $\langle g \rangle = \int_0^1 g(t) dt$. Then $f(t)$ from (1) satisfies the condition $\langle f \rangle = 0$. The case when in addition $\langle v^* \rangle \neq 0$, is of special interest. The average velocity in the terminal rate differs then from zero, i.e. directed drift occurs. Let us try to estimate the value of the direct component for the precise solution $v^*(t)$ starting from the values of the direct component for the outputs of the successive iterations. First of all $\langle f \rangle = 0$ implies $\langle v_0^* \rangle = 0$. Integrating by t the expressions for $v_n^*(t)$, we obtain the formulae $\langle v_n^* \rangle = (\epsilon/\lambda) \langle G(v_{n-1}^*) \rangle$, $n = 1, 2, \dots$. Let us assume that

$$\langle v_1^* \rangle \neq 0 . \tag{5}$$

Then the direct component value for the precise solution satisfies the estimate

$$|\langle v^* \rangle - \langle v_1^* \rangle| \leqslant \frac{A\epsilon/\lambda}{1 - A\epsilon/\lambda} \langle |\Delta_1^*| \rangle, \tag{6}$$

where

$$\Delta_1^*(t) = \frac{\epsilon}{e^\lambda - 1} \int_0^1 e^{\lambda\theta}\, G(v_0^*(t+\theta))\, d\theta .$$

If $(A\epsilon/\lambda)\,[(1 - A\epsilon/\lambda)]^{-1} \langle |\Delta_1^*| \rangle < \langle |v_1^*| \rangle$, then (5) and (6) imply that the precise solution has a direct component whose sign coincides with the sign of $\langle v_1^* \rangle$, and the absolute value is estimated by (6).

As it is stated above, NEFP occurs when $\langle f \rangle = 0$ but $\langle v^* \rangle \neq 0$. If the nonlinear friction component $\epsilon G(v)$ differs from zero, the required situation is rather the rule than the exception. We, therefore, mention the conditions which contradict this and which eliminate the possibility of NEFP. A function $g(t)$ is called antiperiodic if there exists such a $T > 0$ that the equality $g(t + T) = -g(t)$ holds for each $t \in R$. If $f(t)$ in eq. (1) is an antiperiodic function then $\langle v_1^* \rangle = 0$, as well as $\langle v^* \rangle = 0$.

The function $\sin(2\pi t)$, $\cos(2\pi t)$ and their odd powers are antiperiodic ($T = \frac{1}{2}$), thus a field which alters according to one of those laws cannot induce NEFP. However the saw-shaped signal

$$f(t) = at, \quad t \in [-\tfrac{1}{2}; \tfrac{1}{2}],$$

$$f(t+1) = f(t), \quad t \in R, \tag{7}$$

or a signal consisting of the basic tone and the first overtone

$$f(t) = a\cos(2\pi t) + b\cos(4\pi t + \psi) \tag{8}$$

provide (5). For $G(v) = v^3$

$$\langle v_1^* \rangle = \tfrac{7}{3}\, \epsilon a/\lambda^5 + o(1/\lambda^5)$$

for the saw-shaped signal case (7) and

$$\langle v_1^* \rangle = \frac{\epsilon}{\lambda} \frac{a^2 b}{2(\lambda^2 + 4\pi^2)(\lambda^2 + 16\pi^2)^{1/2}}$$

for the sum of two harmonics (8). From the aforesaid it follows that for smal enough ϵ the precise solution has also a direct component, i.e. NEFP occurs.

4. Let us compare the effectiveness of the NEFP and that of electrophoresis in a constant field. We consider the constant force field $F = \langle |f| \rangle$ along $f(t)$. Under the action of F a particle in the terminal rate will drift with some constant speed $\mathcal{V}$. We define the effectiveness of NEFP as the quotient $\langle v^* \rangle / \mathcal{V}$. In the saw-shaped signal case (7) or the case of the sum of two harmonics (8) the value of $\langle v^* \rangle$ is estimated from below according to (6). In this connection the effectiveness is not less than several percent.

In conclusion we note that the terminal solution to eq. (1) depends on the signal form of the applied force in a complicated way. Signals with equal amplitudes but of diverse forms may give an absolutely different effectiveness of NEFP. In this sense it can be stated that the nonlinear medium (dispersed particles in a non-newtonian fluid) is sensitive to information characteristics of the action as well as to its power ones. In particular the equation with a complete friction force proportional to the one-third power of the velocity has been considered along with eq. (1). The expression $\lambda v^{1/3}$ for friction velocity dependence is in qualitative agreement with the experimental data for electro-osmosis by a small strength electric field in water [2]. In this case it appears to be possible to pick out the signal $f(t)$ as a sum of six successive harmonics in such a way that the effectiveness reaches 43%.

References

[1] The Encyclopedia of Electrochemistry (Chapman and Hall, London, 1964).
[2] S.S. Doochin and B.V. Derjagin, Elektrofores (Nauka, Moscow, 1976).
[3] V.N. Shilov and V.R. Estrela-Ljopis, in: Surface forces in thin films and dispersed systems (Nauka, Moscow, 1972) p. 115.
[4] M. Reiner, Deformation, strain and flow (Lewis, London, 1960).
[5] N.F. Bondarenko, Dokl. Akad. Nauk. SSR 177 (1967) 383.
[6] H.A. Pohl, in: Coherent exitations in biological systems (Springer, Berlin, 1983) p. 199.
[7] A.M. Lyapunov, General problem of movement stability (ONTI, Moscow, 1935).