Firing statistics of a neuron with delayed feedback inhibition stimulated with a renewal point process

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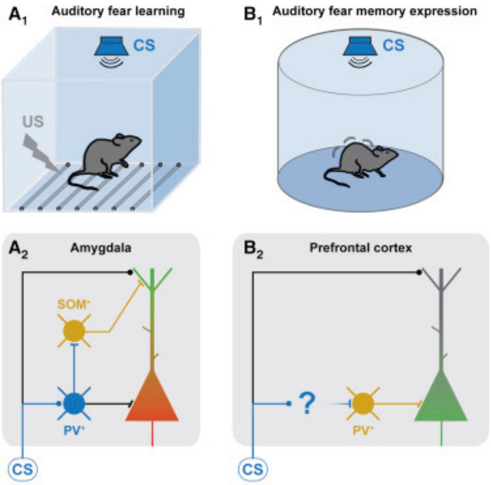
Motivation: cortical disinhibition



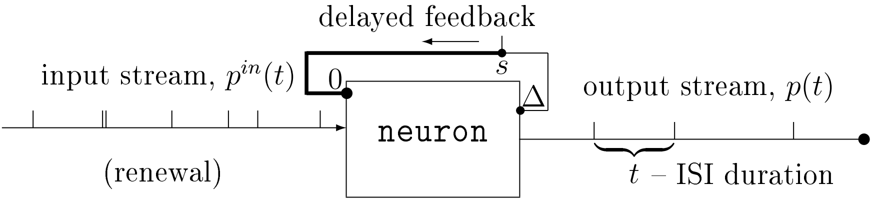
Cortical disinhibition — the transient ceasing of inhibition — is important for some cognitive processes, for instance, learning and memory.

Letzkus, J., et al.: Disinhibition, a Circuit Mechanism for Associative Learning and Memory. Neuron 88, 264–276 (2015)

For PV interneurons the main source of inhibition is autaptic transmission.

Deleuze, C., et al.: Strong preference for autaptic self-connectivity of neocortical PV interneurons facilitates their tuning to γ-oscillations. PLOS Biol. 17, e3000419 (2019)

Neuron with delayed feedback inhibition

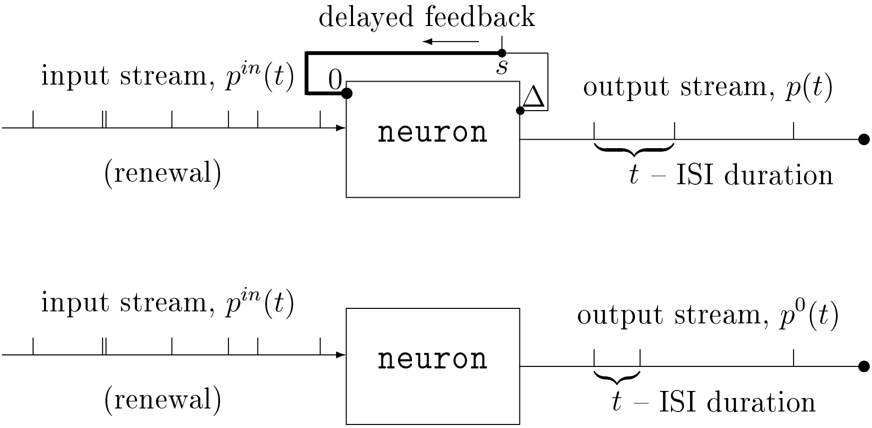


Input and output streams – stochastic point processes

ISI – interspike interval

PDF – probability density function

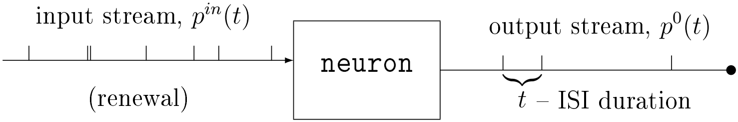
Neuron with delayed feedback inhibition



We have $p^{in}(t)$, $p^0(t)$ and Δ; we're looking for $p(t)$

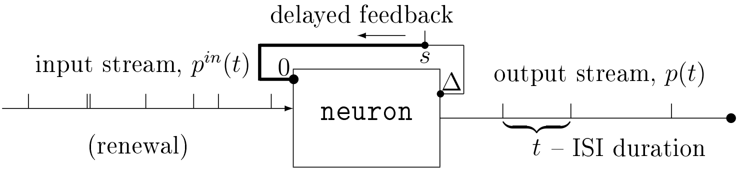
Class of neuronal models

1. Neuron is deterministic.
2. Neuron can be triggered only at the moment of receiving an input impulse.
3. Just after triggering, neuron fires a spike and immediately appears in its resting state, and remains there until an input impulse is received.
4. Neuron is stimulated with a renewal point stochastic process of excitatory impulses with ISI PDF $p^{in}(t)$, t – ISI duration.
5. Output ISI PDF is $p^0(t)$.



Delayed feedback inhibition properties

1. The time delay in the feedback line $\Delta > 0$ is constant.
2. The feedback line is able to convey no more than one impulse.
3. After receiving an impulse from the feedback line, the neuron appears in its resting state, and that impulse is immediately forgotten.

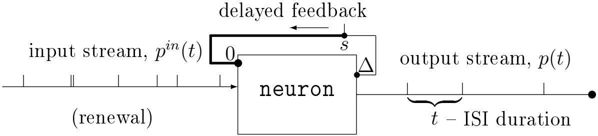


General scheme

$$p(t) = \int_0^{\Delta} ds\, f(s) p(t|s)$$

$f(s)$ — PDF of times-to-live s in the stationary regime

$p(t|s)$ — conditional probability density to get the output ISI of duration t if at the beginning of the ISI there was an impulse in the feedback line with time-to-live s



s — time-to-live

PDF of times-to-live $f(s)$

If the condition

$$\int_0^{\Delta} p^0(s)ds + \Delta \sup_{s\in[0;\Delta]} p^0(s) < 1$$

is met then $f(s)$ in the stationary regime is the solution of the equation

$$f(s) = \int_s^{\Delta} ds'\, p^0(s'-s)f(s') + \delta(s-\Delta)\int_0^{\Delta} ds'\, P^0(s')f(s') = g(s) + a\delta(\Delta - s)$$

$$P^0(s') = 1 - \int_0^{s'} p^0(s)ds.$$

$p(t|s)$

$$p(t|s) = \chi(s-t)p^0(t) + \int_s^t ds''\, p^{o\_if}(t-s'')\times$$

$$\times\left(p^{in}(s'') + \int_0^s ds' \tilde{P}^0(s')p^{in}(s''-s')\right)$$

$$p^0(t) = \int_0^t dt'\, p^{in}(t')p^{o\_if}(t-t')$$

$$P^0(t') = P^{in}(t') + \int_0^{t'} ds'\, \tilde{P}^0(s')P^{in}(t'-s')$$

$$P^{in}(s) = 1 - \int_0^s p^{in}(t)dt,\; P^0(s) = 1 - \int_0^s p^0(t)dt$$

Output ISI PDF $p(t)$: $t < \Delta$

$$p(t) = \int_0^t ds\, g(s) \int_s^t ds''\, p^{o\_if}(t - s'') \Bigg(p^{in}(s'') +$$

$$+ \int_0^s ds'\, \tilde{P}^0(s') p^{in}(s'' - s') \Bigg) + p^0(t) \Bigg(\int_t^\Delta ds\, g(s) + a \Bigg)$$

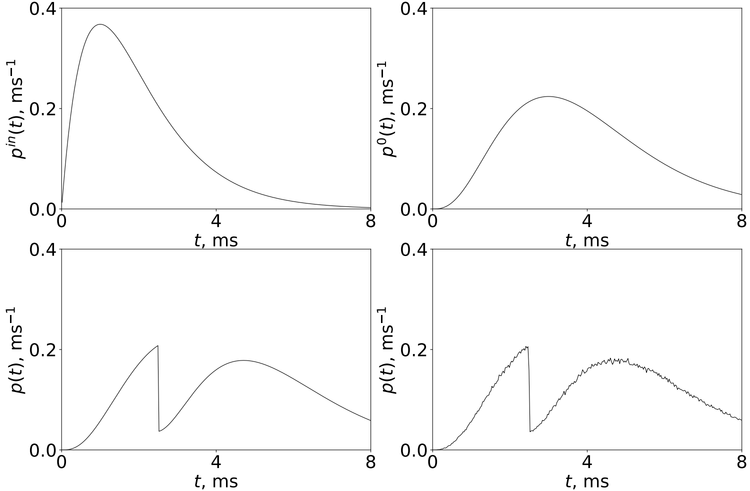
Output ISI PDF $p(t)$: $t > \Delta$

$$p(t) = \int_0^\Delta ds\, g(s) \int_s^t ds''\, p^{o\_if}(t - s'') \times$$

$$\times \left(p^{in}(s'') + \int_0^s ds'\, \tilde{P}^0(s') p^{in}(s'' - s') \right) +$$

$$+ a \int_\Delta^t ds''\, p^{o\_if}(t - s'') \left(p^{in}(s'') + \int_0^\Delta ds'\, \tilde{P}^0(s') p^{in}(s'' - s') \right)$$

Erlang-2 input, binding neuron



$$p^{in}(t) = \lambda^2 t e^{-\lambda t}, \quad p^0(t) = \frac{\lambda^4 t^3}{3!} e^{-\lambda t}$$

$\lambda = 1$ ms^{-1}, $\Delta = 2.5$ ms, $\tau = 8$ ms

Conclusions

1. PDF of ISIs for a neuron with delayed feedback inhibition can be expressed through PDF for an input renewal stream and PDF for the same neuron without feedback.
2. It has a jump-type discontinuity for ISIs of a duration that equals the feedback delay.