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From chaos to clock in reverberating neural net. Case study

Alexander Vidybida[1]   Olha Shchur[1]   Victoria Mochulska[2]

[1]M.M.Bogolyubov Institute for Theoretical Physics, Kyiv, Ukraine

[2]Taras Shevchenko National University of Kyiv
Faculty of Physics, Kyiv, Ukraine

Neural Coding 2021, https://sites.uw.edu/nc2021/

Outline

Chaos example

Controlling chaos in the brain

Steven J. Schiff, Kristin Jerger, Duc H. Duong, Taeun Chang, Mark L. Spano & William L. Ditto

Nature volume 370, pages 615–620 (1994)

Outline

Network used for simulation
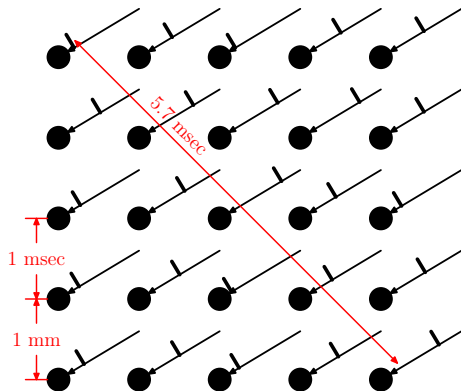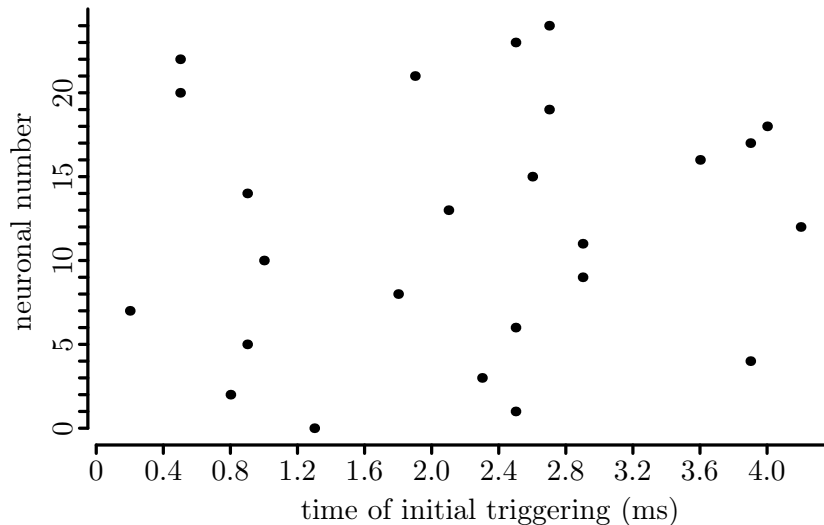

LIF neurons

$V_{th} = 20$ mV,

$h = 0.999$ mV,
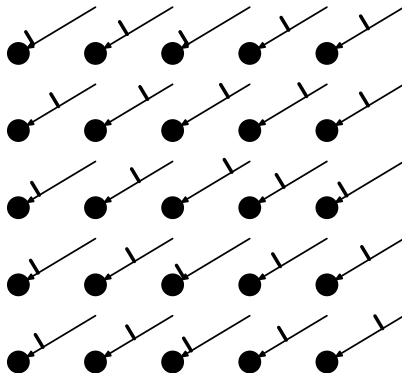
$n_{th} = 21$,

$\tau_M = 20$ msec.

Simulating leaky integrate-and-fire neuron with integers

Vidybida A.K. Mathematics and Computers in Simulation.
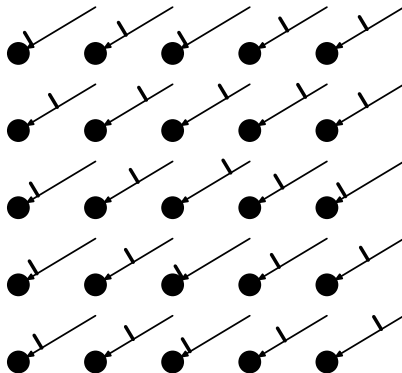159:154-160 (2019)

Single stimulus
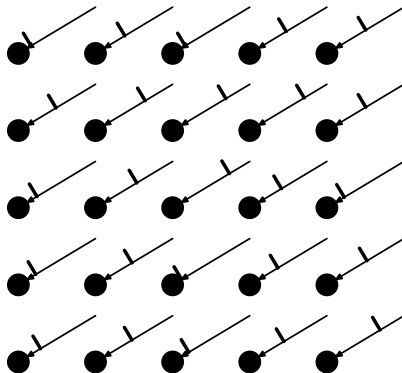
Accepting external stimulus (4 msec long)

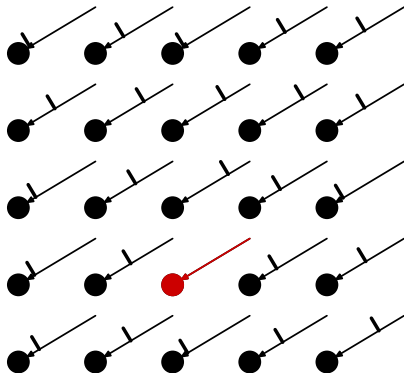$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec
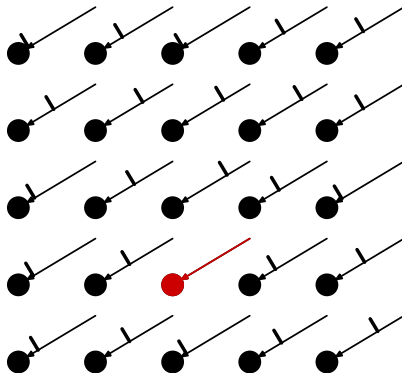
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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$　msec

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$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$$dt = 0.1 \quad \text{msec}$$
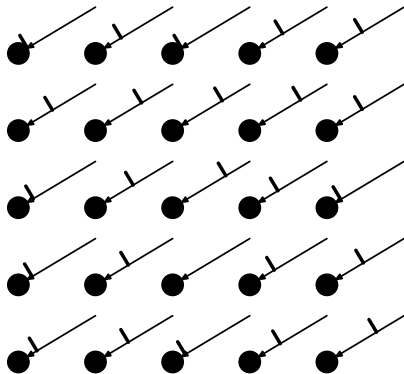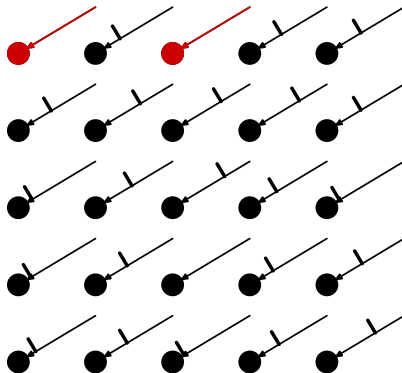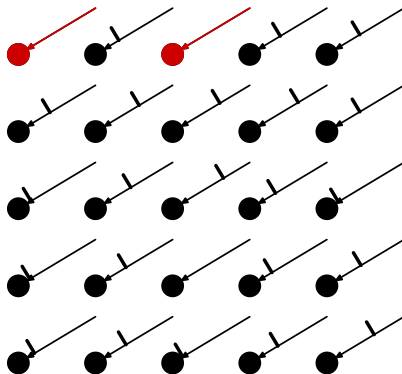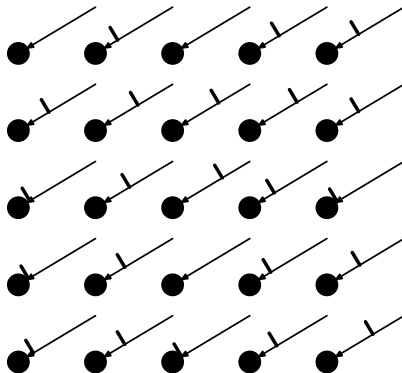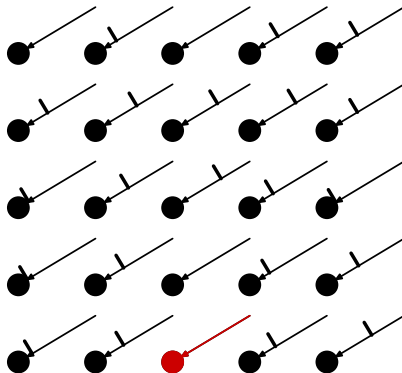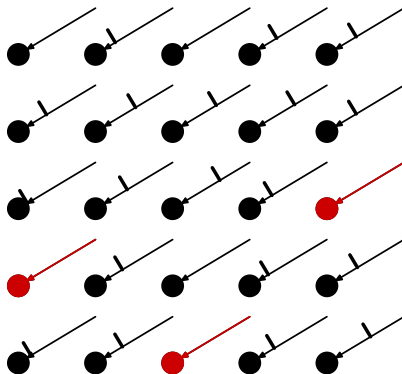
Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec
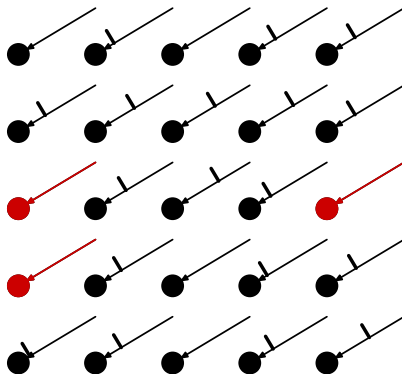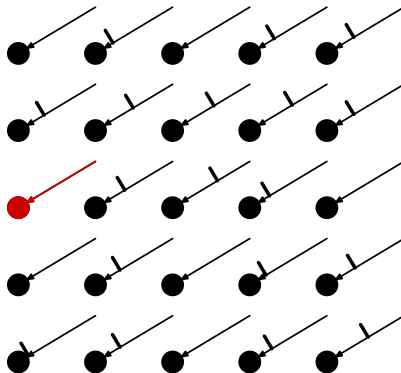
Accepting external stimulus (4 msec long)

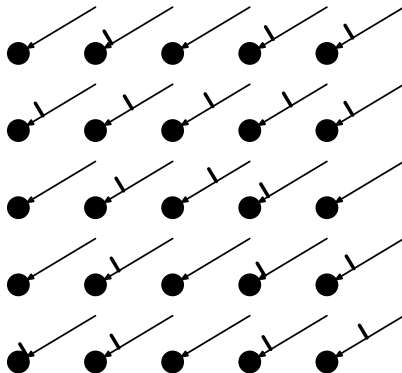$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

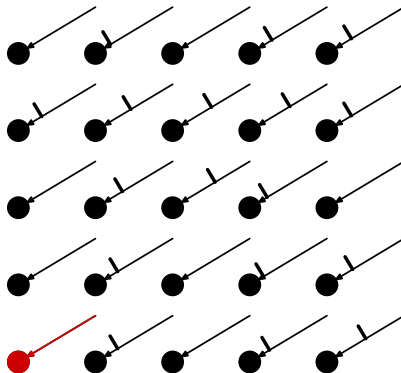**Accepting external stimulus (4 msec long)**

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)
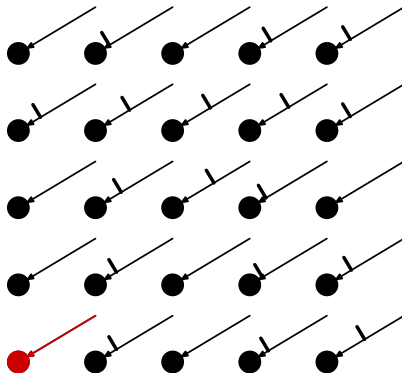

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

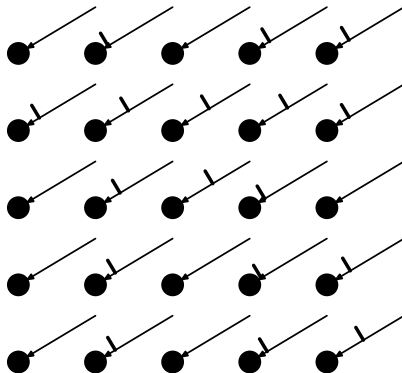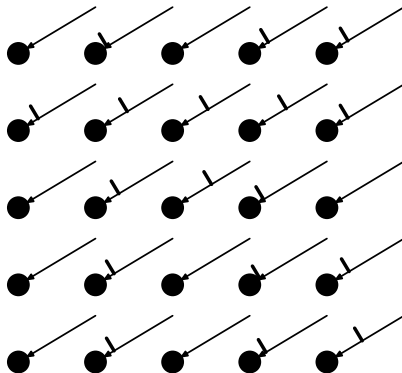$dt = 0.1$ msec

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

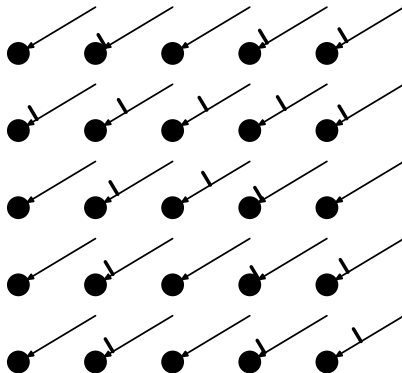**Accepting external stimulus (4 msec long)**

$$dt = 0.1 \quad \text{msec}$$

Accepting external stimulus (4 msec long)

$$dt = 0.1 \quad \text{msec}$$

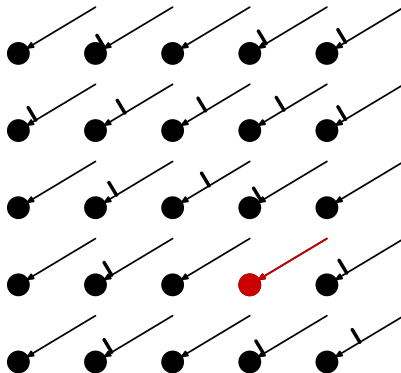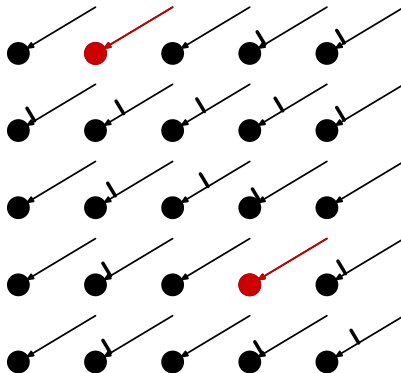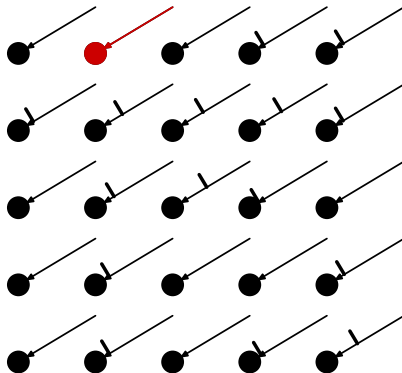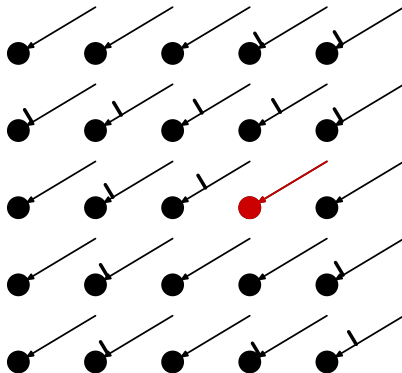**Accepting external stimulus (4 msec long)**
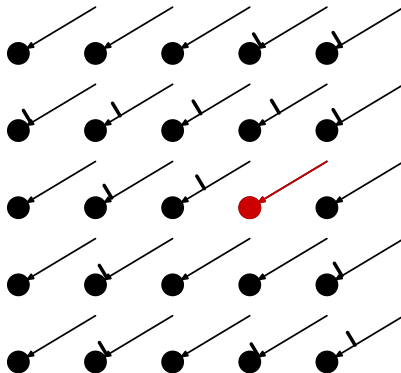

$$dt = 0.1 \quad \text{msec}$$

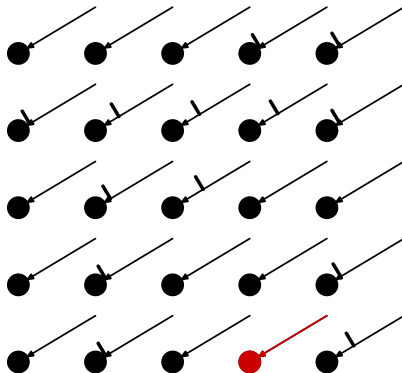## Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

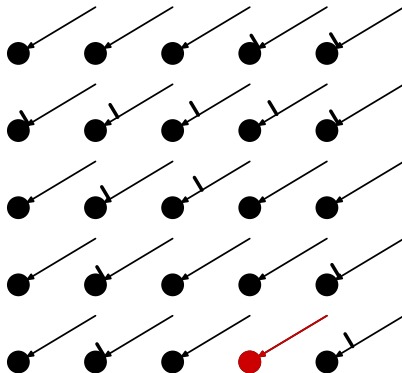**Accepting external stimulus (4 msec long)**

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

$dt = 0.1$ msec

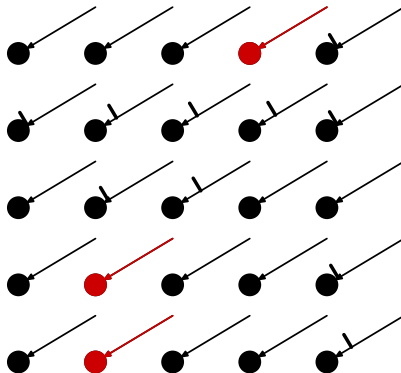**Accepting external stimulus (4 msec long)**

$dt = 0.1$ msec

Accepting external stimulus (4 msec long)

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Accepting external stimulus (4 msec long)

$dt = 0.1$ msec
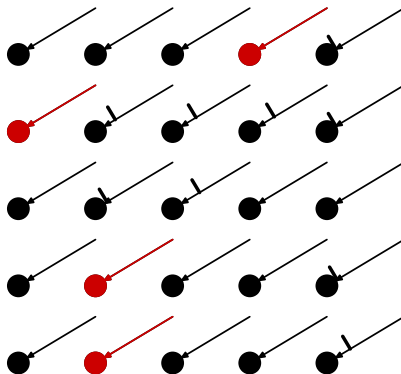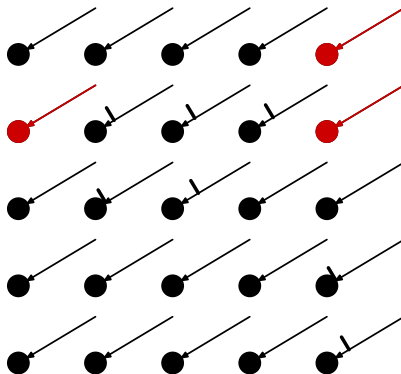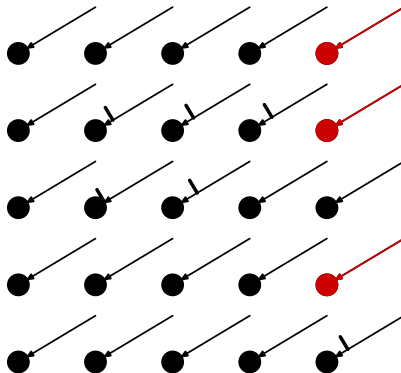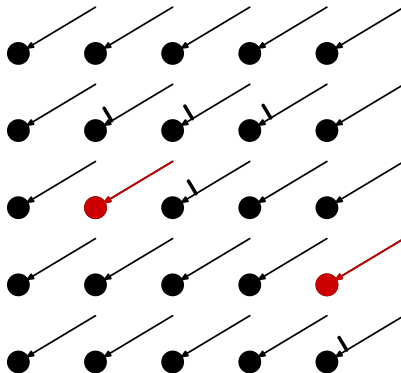
Accepting external stimulus (4 msec long)
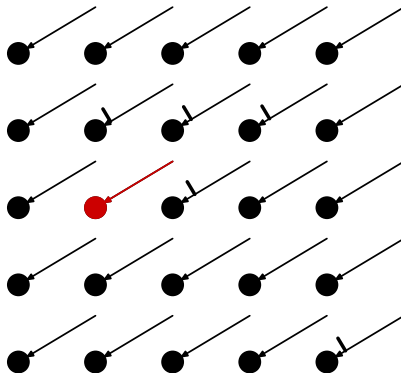

$dt = 0.1$ msec

Dynamic: ISIs: chaotic

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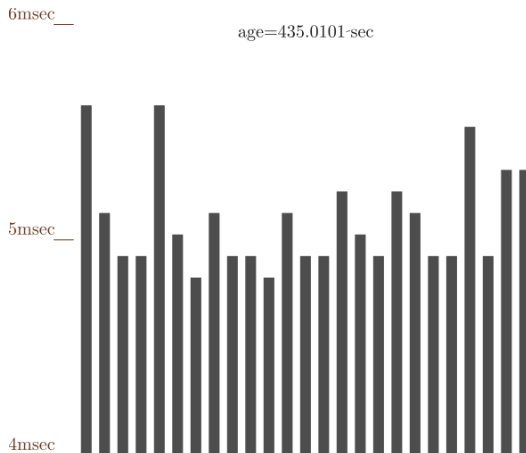
Dynamic: ISIs: periodic

Stable ISIs are 4.8 + 4.8 msec and 4.7 + 4.9
msec, Period = 9.6 msec

Dynamic: Compound voltage, periodic regime

$\sum\limits_{i=1}^{25} V_i$ mV   two periods are shown

Dynamic: Compound voltage, 10 sec earlier

$$\sum_{i=1}^{25} V_i \text{ mV}$$ two periods long interwal is shown
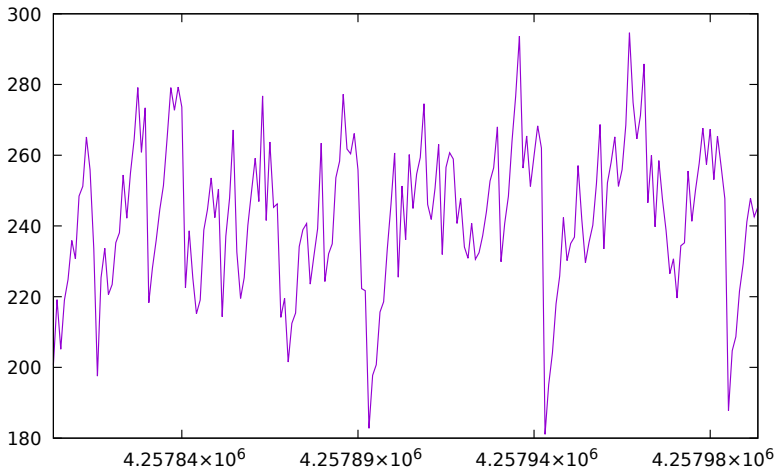
$\leftarrow$ $2 \cdot 9.6$ msec $\rightarrow$

Phase portrait, 20 sec to period

V_{22} mV   one period long interwal is shown

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Phase portrait, 0 sec to period

V_{22} mV

one period is shown
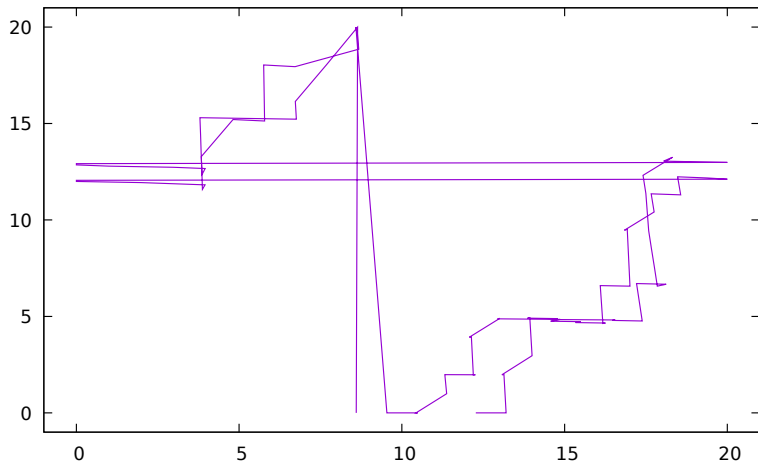
V_{21} mV

Properties

Time to cross diagonal — **5.7 milliseconds**

Stimulus duration — **4 milliseconds**

Time to periodic regime — **435.7805 seconds**

Period length — **9.6 milliseconds**

435780.5 / 5.7=76453

Analysis: 0-1 test for chaos, 1

On the Implementation of the 0–1 Test for Chaos

Gottwald,G.A., Melbourne,I.

SIAM Journal on Applied Dynamical Systems 8(1):129-145 (2009)

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Analysis: 0-1 test for chaos, 2

$$V = \{ V_0, V_1, \ldots, V_{N-1} \}, \quad V_i \in \mathbb{R}^1, \quad i = 0, 1, 2, \ldots N-1, \quad (1)$$

Choose $c \in]0; \pi[$ and calculate a set of 2D points $(p_c(n), q_c(n))$:

$$p_c(n) = \sum_{0 \leq i \leq n} V_i \cos(i \cdot c), \quad q_c(n) = \sum_{0 \leq i \leq n} V_i \sin(i \cdot c). \quad (2)$$

Then compute the asymptotic growth rate Kc of the mean square displacement of $(p_c(n), q_c(n))$ for different c. We then compute the median of these values of Kc for different c to compute the final result K = median(Kc). The test states that a value of K $\approx$ 0 indicates regular dynamic, and K $\approx$ 1 indicates chaotic dynamic.

Analysis: 0-1 test for chaos, 3

Kc,   median = 0.99

c
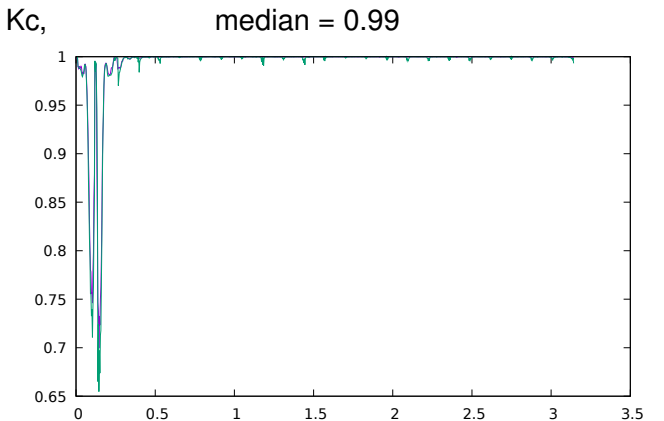
Behavior of Kc for chunks #1 and #9

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Analysis: 0-1 test for chaos, 4

Kc, median = 0.013

c
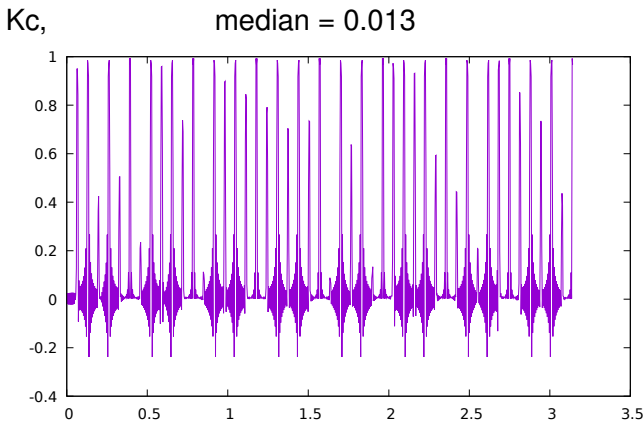
Behavior of Kc for chunk #10

Analysis: Permutation entropy, 1

Permutation Entropy: A Natural Complexity Measure for Time Series

Bandt,Ch., Pompe,B.

Physical Review Letters 88(17):174102 (2002)

Calculating Permutation Entropy without Permutations

Vidybida,A.K.

Complexity 2020:7163254 (2020)

Analysis: Permutation entropy, 2

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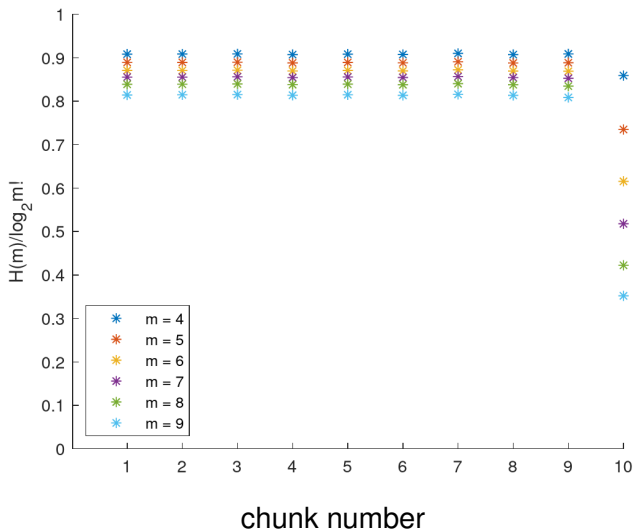
Analysis: Stimulus perturbation

We use the same initial state of the
network.

Instead, we perturb the stimulus.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

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Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

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Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

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Analysis: Perturbed stimuli used
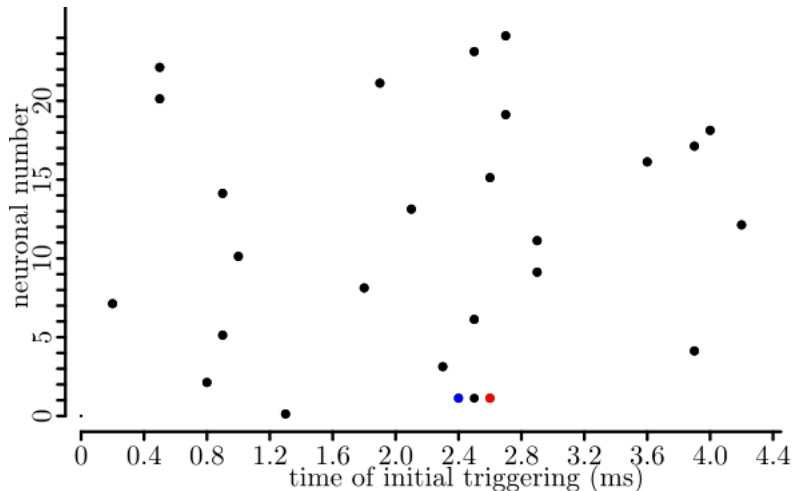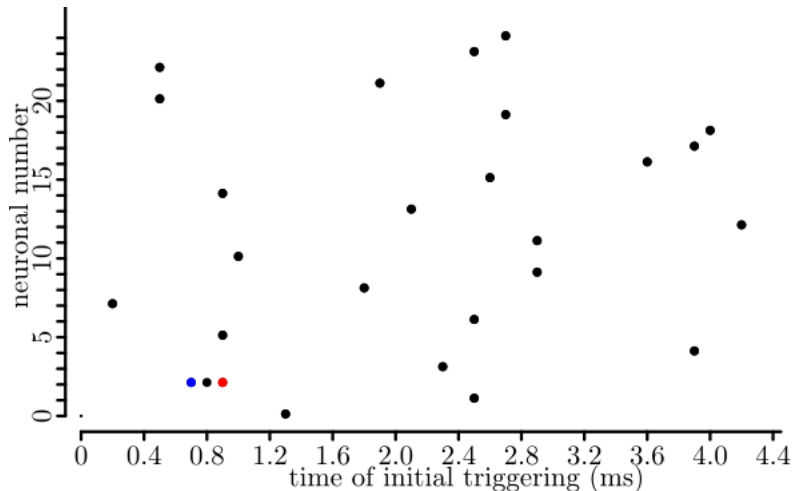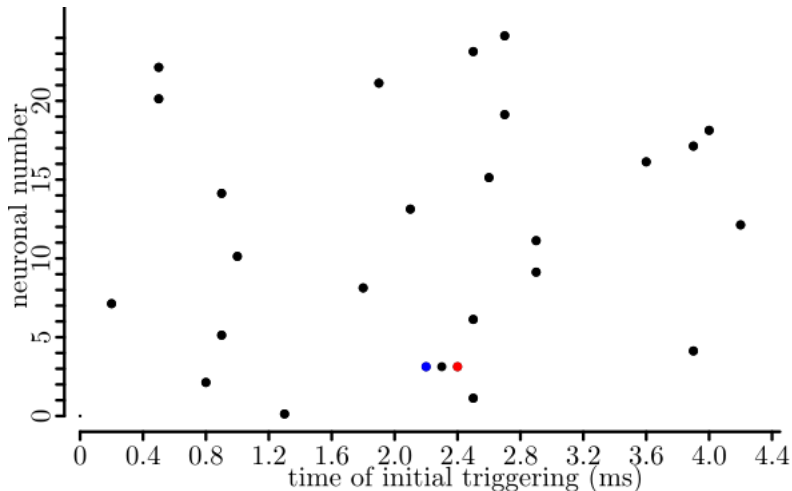

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

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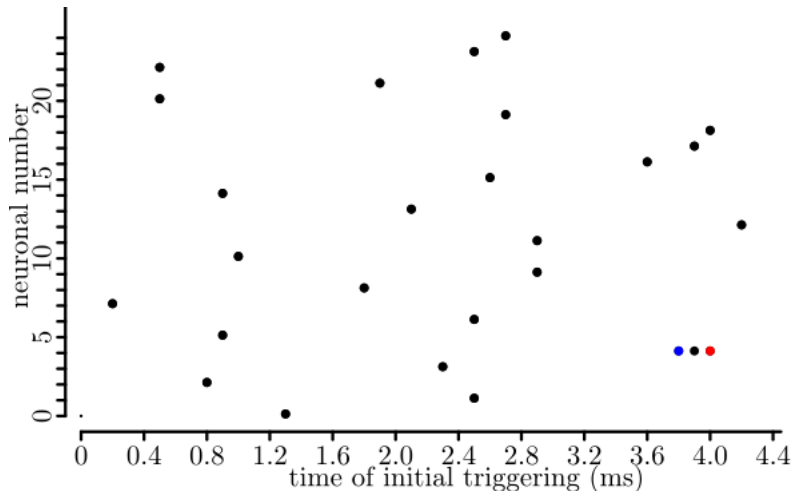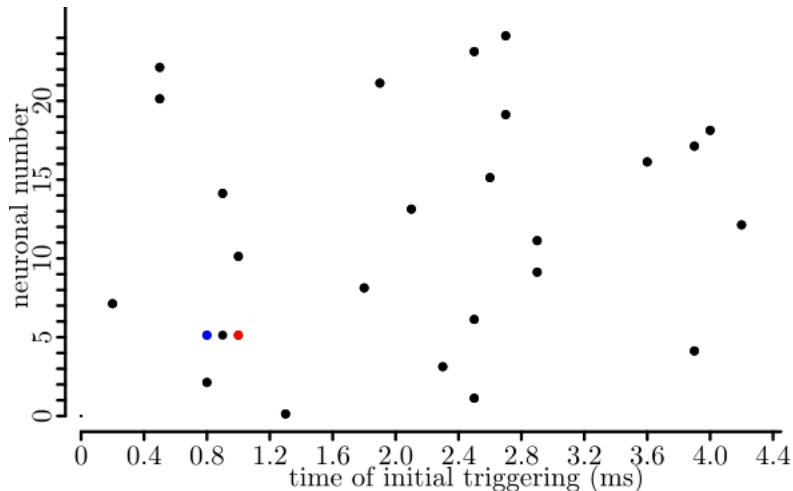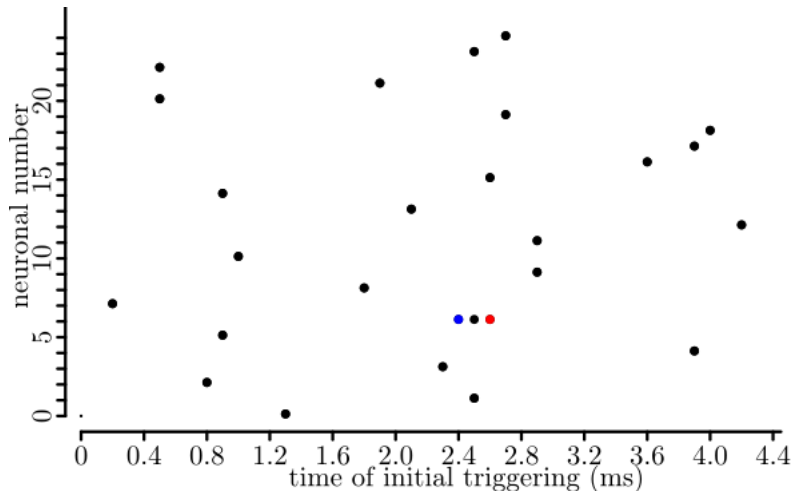
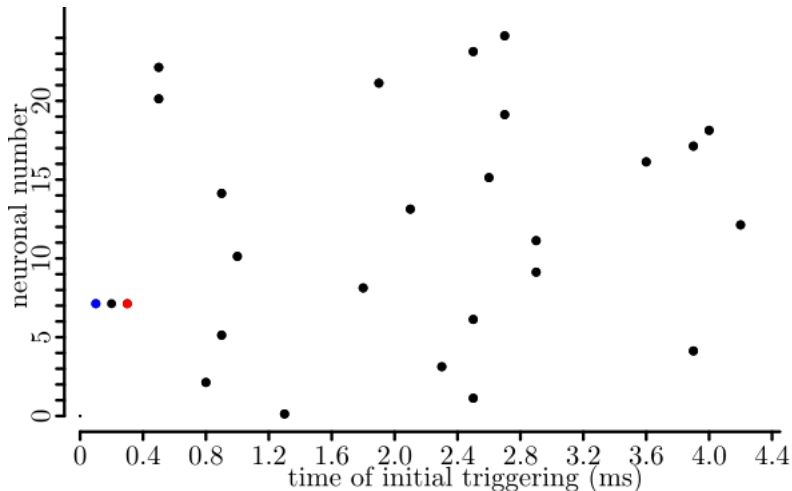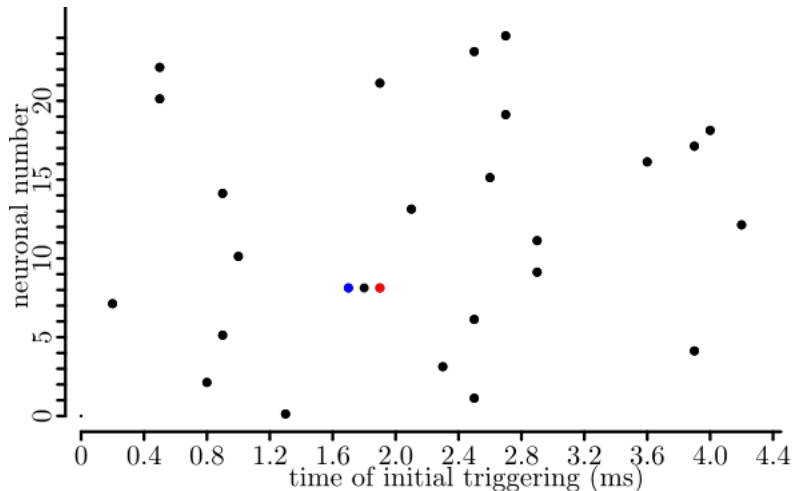## Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

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Analysis: Perturbed stimuli used

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Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

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Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

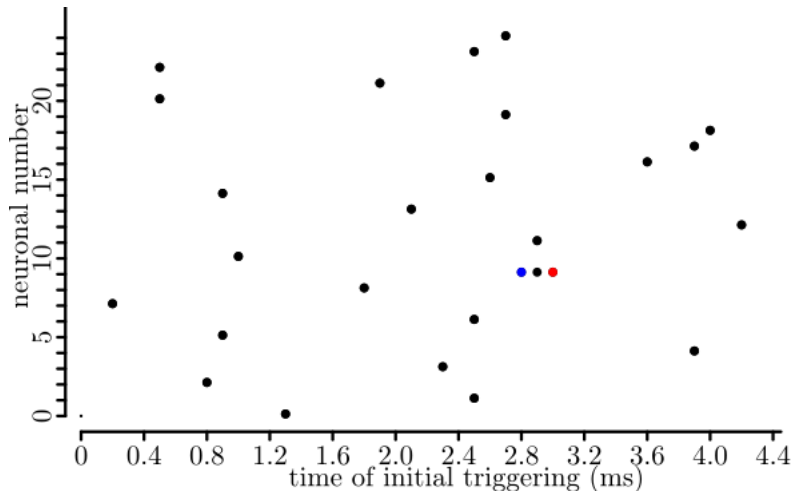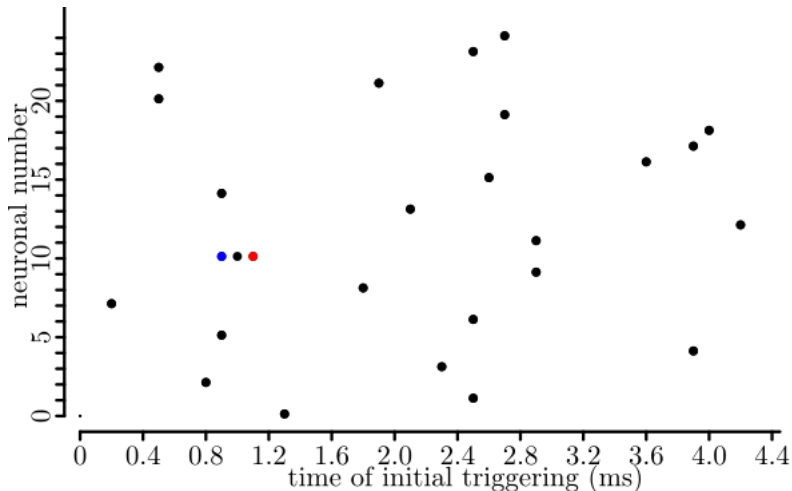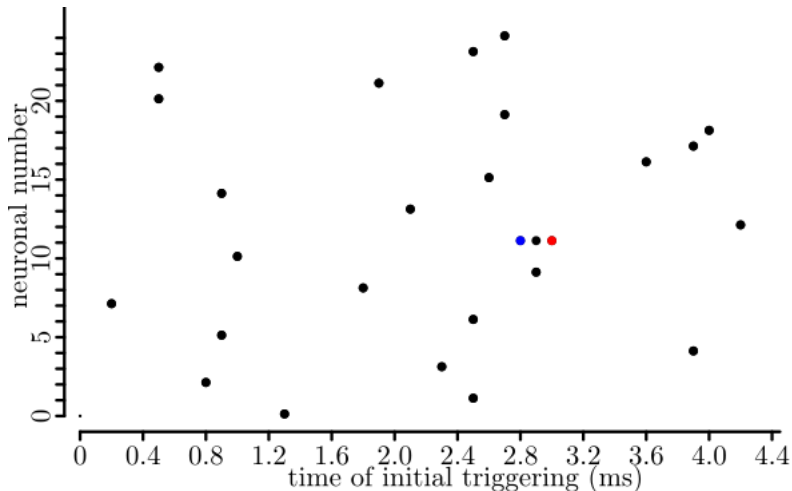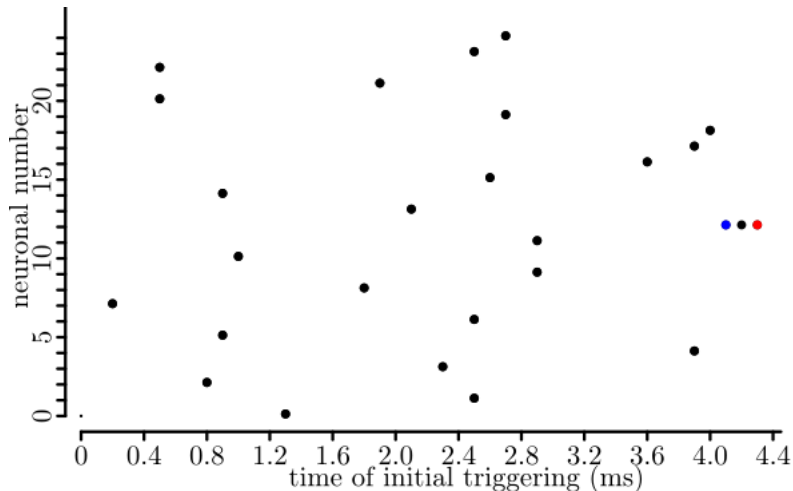## **Analysis: Perturbed stimuli used**

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

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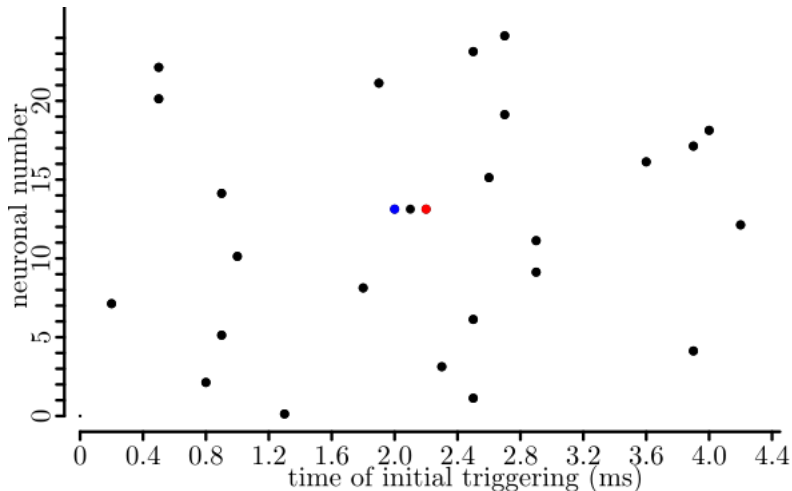o
oo

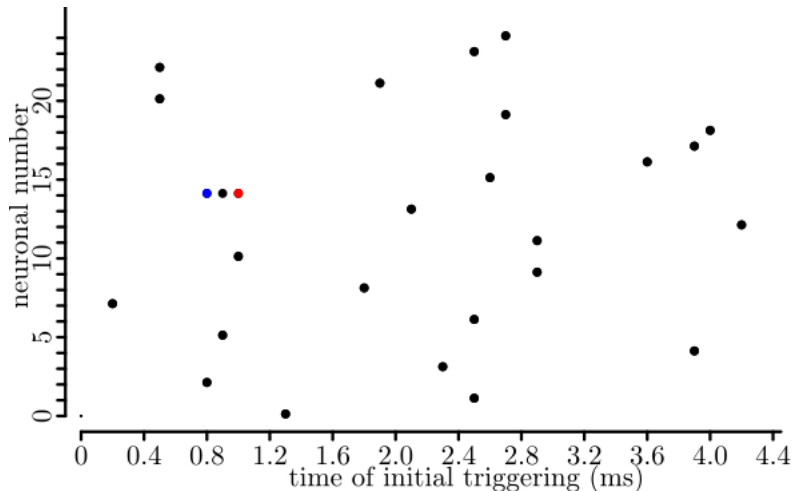## **Analysis: Perturbed stimuli used**

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

Analysis: Perturbed stimuli used

Red dot is for increased delay, blue dot - for decreased.

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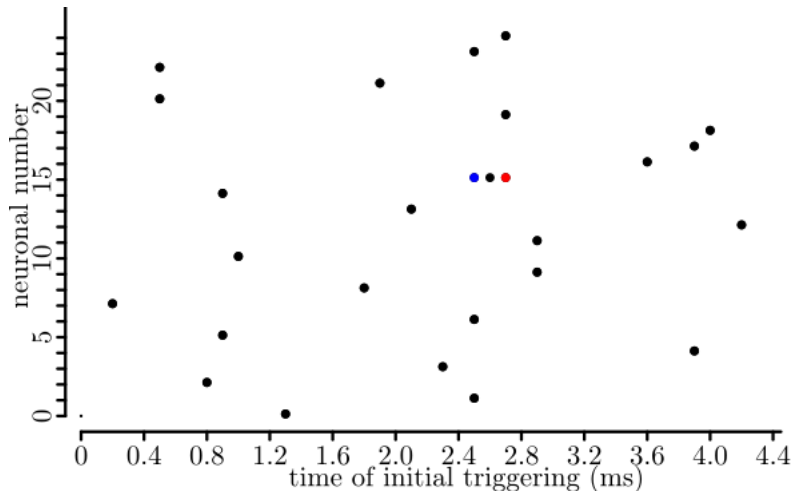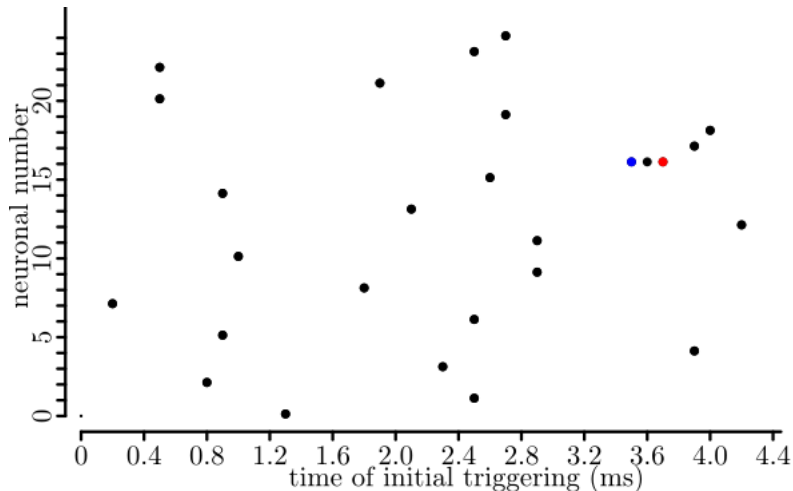## **Analysis: Perturbed stimuli used**

Red dot is for increased delay, blue dot - for decreased.

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Dynamics observed
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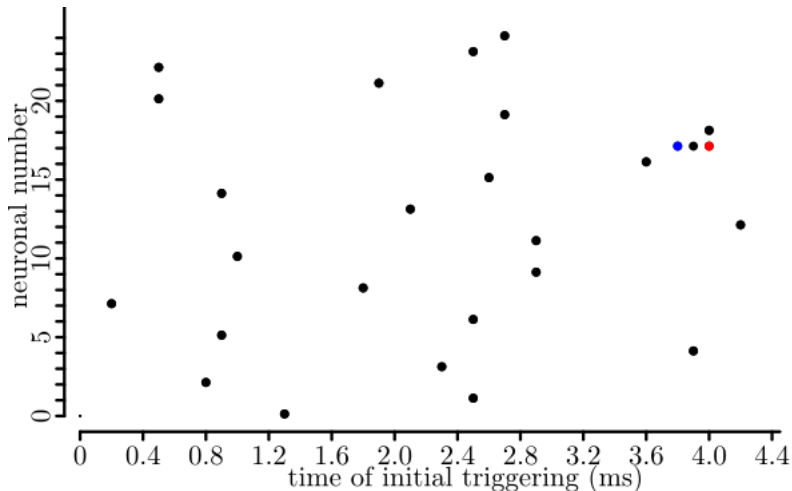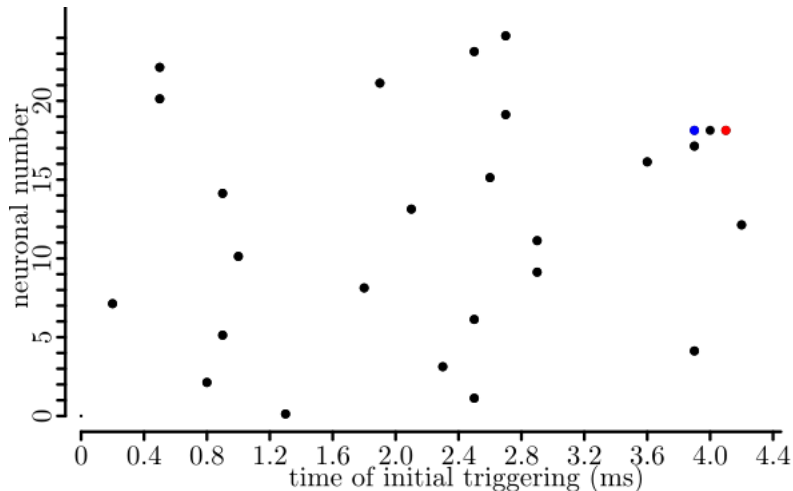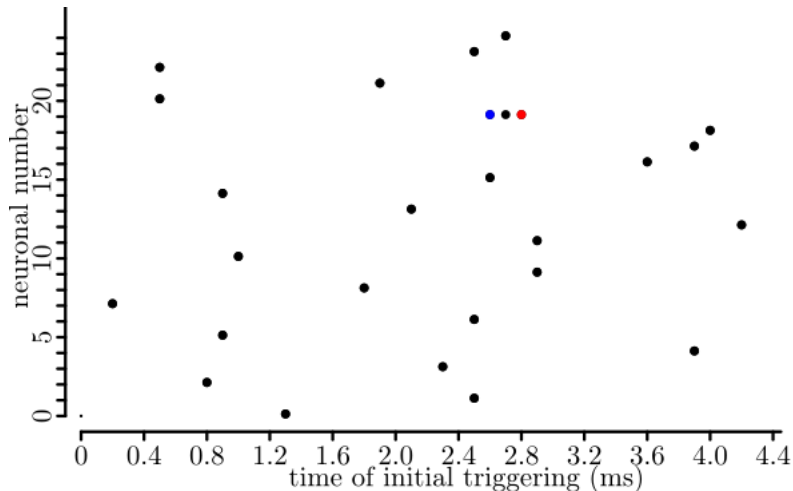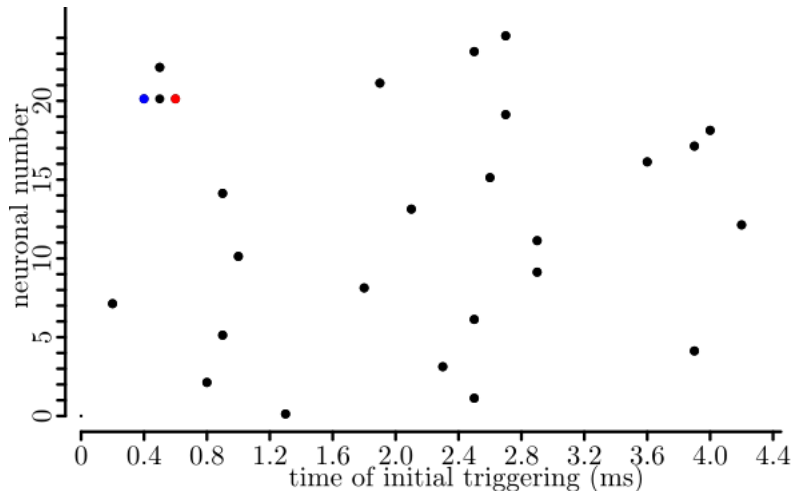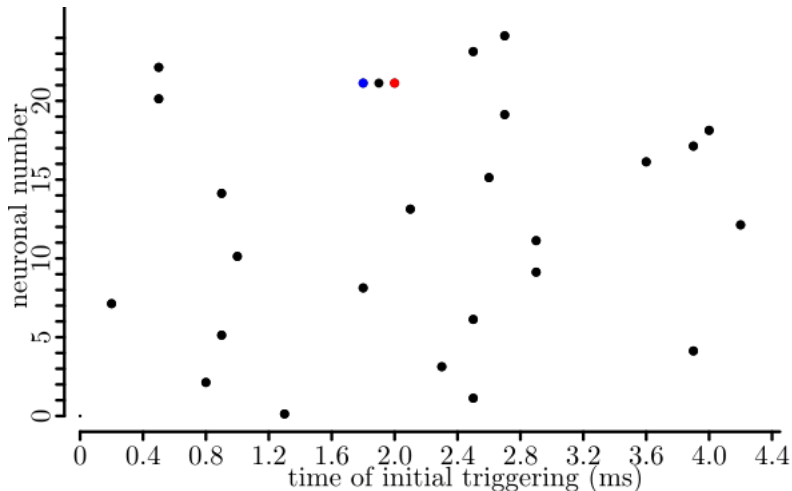## **Analysis: Perturbed stimuli used**

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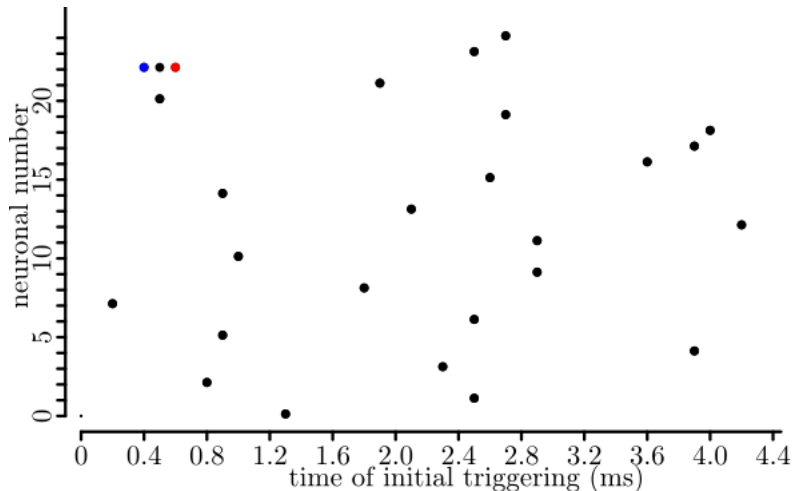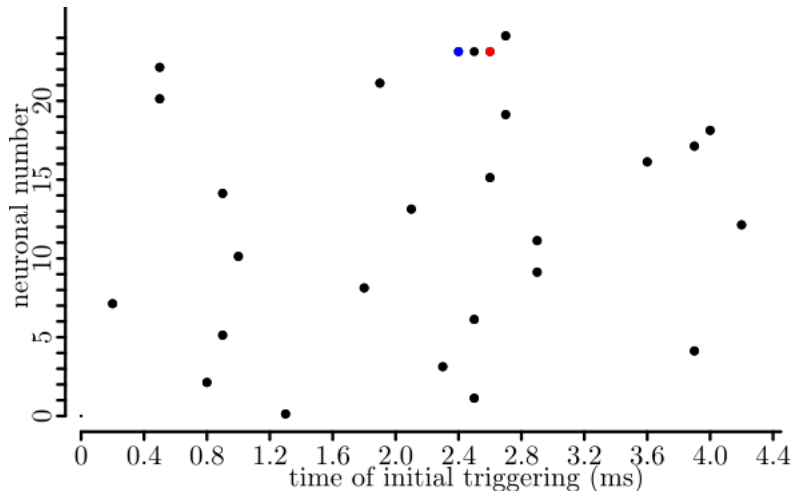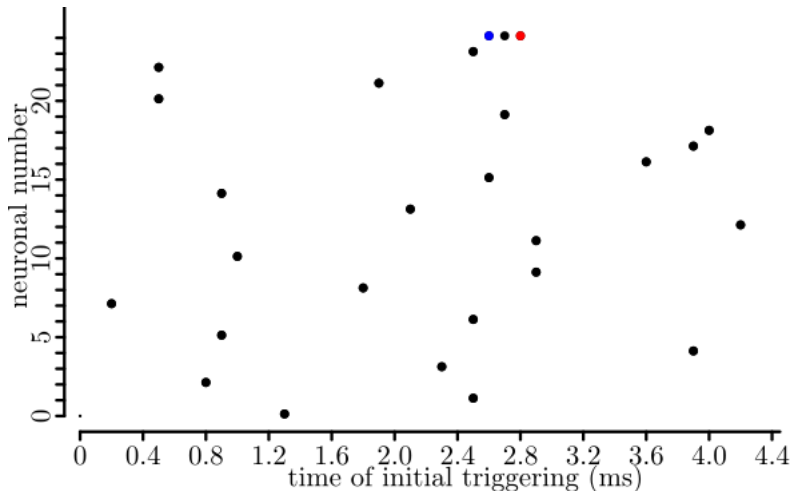
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Analysis: Perturbed dynamics

Period, ms	Number of cases	Relaxation time, s
0 (activity dies)	1	0.14
9.6	32	26.5 — 436
9.7	1	247
10.4	8	73 — 1569 (26 min)
10.5	2	229
10.6	1	302
19.2	1	74
19.5	2	149, 651
41.6	3	64 — 412
	51	

Conclusions

- A small reverberating net can have a complicated dynamic
- The dynamic can be highly sensitive to stimulus perturbation
- A pattern observed in our case: after long period of chaos, the dynamic becomes periodic

Outline

Missed reality

- Our network is fully connected
- Inhibitory neurons are absent
- Stochastic features are absent
- Plasticity/adaptation is absent

QUESTIONS, PLEASE

1. Vidybida A.K. Simulating leaky integrate-and-fire neuron with integers Mathematics and Computers in Simulation. 159:154-160 (2019)

2. Vidybida A.K., Shchur O.V. Information reduction in a reverberatory neuronal network through convergence to complex oscillatory firing patterns BioSystems Neural Coding 2016 161(Supplement C):24-30 (2017)