

Original articles

Simulating leaky integrate-and-fire neuron with integers

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Abstract

The leaky integrate-and-fire (LIF) neuron represents standard neuronal model used for numerical simulations. The leakage is implemented in the model as exponential decay of trans-membrane voltage towards its resting value. This makes inevitable the usage of machine floating point numbers in the course of simulation. It is known that machine floating point arithmetic is subjected to small inaccuracies, which prevent from exact comparison of floating point quantities. In particular, it is incorrect to decide whether two separate in time states of a simulated system composed of LIF neurons are exactly identical. However, decision of this type is necessary, e.g. to figure periodic dynamical regimes in a reverberating network. Here we offer a simulation paradigm of a LIF neuron, in which neuronal states are described by whole numbers. Within this paradigm, the LIF neuron behaves exactly the same way as does the standard floating point simulated LIF, although exact comparison of states becomes correctly defined.

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1. Introduction

Leaky integrate-and-fire (LIF) neuron, [6,9] is a universally-accepted “standard” neuronal model in theoretical neuroscience. Usage of LIF as a model allowed obtaining numerous theoretical results, see e.g. [1,2,7,10].

When input impulses are absent, the membrane potential of the LIF model decays exponentially. In computer simulations, this makes inevitable the usage of machine floating point arithmetic. However, the floating point calculations can be considerably inaccurate in some cases, see e.g. [3,4]. These cases are not so frequently occurring. Therefore, if one studies neuronal firing statistics, when a simulation is repeated many times with slightly different parameters, the errors due to floating point pitfalls can be negligible small.

The problem arises if one needs to check if two neuronal states obtained in the course of simulation are identical. This kind of check is necessary, e.g. to figure periodic dynamical regimes of a reverberating network, see [11] for an example. In order to check if a neural net is in a periodic regime, one needs to check whether two states the net passes through at two distinct moments of time are exactly the same. If states of individual neurons the net is composed of are described by floating point variables, then this expects checking that two floating point numbers are exactly equal — the operation, which is not permitted in programming due to small, but inevitable inaccuracy adhered to the machine floating point arithmetic. Instead, the option is to check whether the distance between two numbers is less than a

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”machine epsilon”, [5]. This does not help to find periodic regimes reliably. Indeed, in this comparison paradigm, two different states may happen to be treated as identical and this may bring about a fake periodic regime. Taking into account that rounding mode depends on operating system and on machine architecture, it will be difficult to treat in a consistent manner sustained behavior of a dynamical system prone to instability (as do neuronal nets) if simulation is made in floating point numbers.

This problem is avoided in [11], where the binding neuron model was used as neuronal model. The binding neuron states are naturally described by integers, and machine arithmetic of integers is perfectly accurate. This admits a routine check whether two numbers are exactly equal, and finally, whether two states of a neuronal net are identical, provided those states are expressed in terms of whole numbers, exclusively. It would be nice to have the same possibility when a LIF model is used for simulations.

In traditional computer simulations of LIF model driven with input stream of stimulating impulses, a dynamical system is simulated, which has states described with machine floating point numbers. We denote this dynamical system as fpLIF. The purpose of this paper is to offer an approximation, which replaces the fpLIF with intLIF — a dynamical system whose states are expressed in terms of integers. In what follows, we describe the approximation itself and check its quality by running both fpLIF and intLIF with the same stream of input impulses randomly distributed in time. It appears that it is possible to choose the approximation parameters in such a way, that dynamics of both fpLIF and intLIF are exactly the same, if considered in terms of spiking moments.

2. Methods

2.1. The fpLIF model

The simplest LIF model is considered. Namely, the neuronal state at moment t is described as membrane voltage, $V(t)$. The resting state is defined as $V(t) = 0$. Any input impulse advances membrane potential by h , where $h > 0$. Between two consecutive input impulses, $V(t)$ decays exponentially:

$$V(t + s) = e^{-s/\tau} V(t)$$

where $s > 0$, τ — is the membrane relaxation time. Suppose that the threshold value for the LIF neuron is V_0 . The neuron fires a spike every time when $V(t) \geq V_0$, and $V(t)$ is reset to zero after that. The set of possible values of $V(t)$ is the following interval:

$$V(t) \in [0; V_0[\tag{2.1}$$

The above mentioned properties of the LIF model can be routinely coded with $V(t)$ and h declared as floating point quantities. In this case, possible values of $V(t)$ will be those machine floating point numbers, which fall into the interval (2.1). And this gives the fpLIF dynamical system.

2.2. The intLIF model

2.2.1. Pure decay dynamics

In numerical simulation of a dynamical system, the time is advanced in discrete steps, having duration of a small fixed¹ time-step, dt . This gives an approximation of the continuous time t with discrete moments:

$$\mathbb{T} = \{0, dt, 2 dt, 3 dt, \dots\} \tag{2.2}$$

Due to this fact, the membrane voltage, $V(t)$, also changes in a discrete manner from step to step. As a result, in a single run of the pure decay dynamics (without input stimulation) the $V(t)$ will pass through only some discrete values, missing the intermediate ones. Those discrete values can be chosen as an approximation of the continuous interval (2.1). It is clear that the set of the discrete values mentioned depends on the initial value of V . To be concrete, let us chose the following approximating set:

$$\mathbb{V}_\alpha = \{\alpha V_0, \alpha^2 V_0, \alpha^3 V_0, \dots\}, \text{ where } \alpha = e^{-dt/\tau} \tag{2.3}$$

¹ See Discussion for a remark about adjustable dt .

This induces the following representation of the interval (2.1):

$$[0; V_0[= \{0\} \cup [\alpha V_0; V_0[\cup [\alpha^2 V_0; \alpha V_0[\cup \dots \quad (2.4)$$

Any value $V(t) \in [0; V_0[$ falls for some n into interval $[\alpha^{n+1} V_0; \alpha^n V_0[$ and we chose its left end as an approximation for that $V(t)$. The error of this approximation is less than

$$\Delta V_0 = (1 - \alpha)V_0 \quad (2.5)$$

Now, if neuron has membrane potential $V(t) = V \in [\alpha^{n+1} V_0; \alpha^n V_0[$ and one intends to describe by values from \mathbb{V}_α its consequent dynamics due to pure decay, the following should be done. The $V(t)$ value should be replaced with $V_d = \alpha^{n+1} V_0$, and the subsequent decay dynamics simply propels $V(t)$ through values $\alpha V_d, \alpha^2 V_d, \alpha^3 V_d, \dots$ from \mathbb{V}_α . The state of neuron at each time step can be labeled with integer n by the following way: The state $V(t) = \alpha^{n+1} V_0$ obtains label n , where $n \geq 0$. Now, the decay dynamics can be expressed in terms of n . Namely, if at some discrete moment of time $t \in \mathbb{T}$ a state is labeled as n , then the state at the next moment, $t + dt$, is labeled as $n + 1$. Thus, having in mind the correspondence: $n \leftrightarrow \alpha^{n+1} V_0$, the pure decay dynamics becomes as simple as adding unity to the state label at each time step.

For computer simulation purpose, it should be noticed, that the set \mathbb{V}_α is finite. If the state label n is declared in a program as `int`, then \mathbb{V}_α has `INT_MAX + 1` elements, where `INT_MAX` is the largest value of `int`-type variable which can be represented in the operating system. Thus, with state labels of type `int`, instead of (2.3), one has to chose as \mathbb{V}_α the following set of `INT_MAX + 1` elements: $\mathbb{V}_\alpha = \{\alpha V_0, \alpha^2 V_0, \dots, \alpha^{\text{INT_MAX}} V_0, 0\}$, where the value 0 is added to describe the resting state, which is attained just after firing. Consequently, (2.4) should be replaced with the following:

$$[0; V_0[= [\alpha V_0; V_0[\cup [\alpha^2 V_0; \alpha V_0[\cup \dots \cup [0; \alpha^{\text{INT_MAX}} V_0[\quad (2.6)$$

In a 64-bit OS, `INT_MAX = 2147483647`. This imposes a limit on possible duration of the pure decay evolution represented with `int` type labels. If the time step $dt = 0.01$ ms, then `INT_MAX · dt` ~ 350 min. Thus, description of neuronal state by `int` type label fails, if a LIF neuron starts, e.g. with $V(t)$ close to V_0 and does not receive excitatory stimulation for longer than 350 min of real time.

In real networks, stimulus-free period of a neuron involved in a useful task cannot be so long. Indirectly, a very crude upper bound for possible duration of stimulus-free period can be estimated from duration of suppression of activity observed in some brain networks, [8]. Such a suppression can last up to 1000 ms. Based on this value, let us expect that our neuron receives excitatory input impulses² of amplitude h with mean rate higher than $r \sim 0.1$ Hz. Before the first input impulse, neuron is in the state “empty” with $V = 0$. After the first input impulse the state label $n_1 \sim \log_\alpha(h/V_0)$ and this value is small if compared with `INT_MAX`. For example, for $dt = 0.01$ ms, $\tau = 20$ ms, $h/V_0 = 0.01$ one obtains $n_1 \sim 10^4$. The label n_1 gets increment 1 after each time step dt until the next input impulse comes. The mean waiting time for the next stimulation, if expressed in the dt units, is $1/(r \cdot dt) = 10^6$. This means that at the moment of next stimulation, the state label $n_2 \sim n_1 + 10^6 \ll 10^7$. After the next input impulse, the state label $n_3 < n_1$. This suggests that states with labels $n > 10^7$ will be observed quite rare with no chances to attain n value close to the `INT_MAX` provided a neuron receives at least a moderate stimulation.

2.2.2. Input impulse

Suppose that input impulse at moment t advances by h the membrane voltage $V(t)$. If $V(t) \in \mathbb{V}_\alpha$, then $V(t) + h \in \mathbb{V}_\alpha$ does not hold in most cases. One needs to approximate $V(t) + h$ by a suitable value from \mathbb{V}_α , as described in n. 2.2.1, and then to proceed with pure decay dynamics expressed in integers, as described in n. 2.2.1, until the next input impulse arrives.

Preliminary tests of this scheme were performed with LIF neuron stimulated with Poisson stream of input impulses. The standard floating point LIF (fpLIF) simulation and integer LIF (intLIF) simulation, as described here, were performed with the same input streams. The threshold manner of triggering gives chances that firing moments of both models will be the same, if expressed in whole number of dt . Computer simulations performed show that the firing moments of both models are indeed the same for some initial period of time after which they become different. This is because the approximation error, when approximating LIF state (voltage) by a value from \mathbb{V}_α , builds up too fast with each input impulse. In order to decrease the error, one needs more precise approximation of the continuum set $[0; V_0[$, than that given by the discrete set \mathbb{V}_α . The required approximation can be achieved by introducing second order bins into the set \mathbb{V}_α .

² See the next section for exact treatment of input impulses.

2.2.3. Second order bins

Let us choose an integer $N > 1$ and divide each bin from (2.6) to N equal second order bins.³ This gives representation of n th first order bin:

$$[\alpha^{n+1}V_0; \alpha^n V_0[= \bigcup_{i=0}^{N-1} [\alpha^{n+1}V_0 + i \cdot c_n; \alpha^{n+1}V_0 + (i + 1)c_n[$$

where c_n denotes the size of second order bin within the n th first order bin $[\alpha^{n+1}V_0; \alpha^n V_0[$:

$$c_n = (\alpha^n V_0 - \alpha^{n+1} V_0)/N \tag{2.7}$$

Now, if $V(t) \in [\alpha^{n+1}V_0 + i \cdot c_n; \alpha^{n+1}V_0 + (i + 1)c_n[$, then we chose $V_d = \alpha^{n+1}V_0 + i \cdot c_n$ as its approximation. This results in the new set $\mathbb{V}_{\alpha,N}$ of possible values for $V(t)$:

$$\mathbb{V}_{\alpha,N} = \{0\} \cup \bigcup_{n=0}^{\text{INT_MAX}-1} \bigcup_{i=0}^{N-1} \{\alpha^{n+1}V_0 + i \cdot c_n\}$$

Any point, except for 0, in the set $\mathbb{V}_{\alpha,N}$ is labeled with two integers, $\{n, i\}$, where $0 \leq n < \text{INT_MAX}$, $0 \leq i < N$. Any label $\{n, i\}$ corresponds to the membrane voltage $V_{n,i}$ from $\mathbb{V}_{\alpha,N}$: $V_{n,i} = \alpha^{n+1}V_0 + i \cdot c_n$, or:

$$V_{n,i} = \alpha^n V_0 \left(\alpha + \frac{i}{N}(1 - \alpha) \right) \tag{2.8}$$

From the last, it is clear that pure decay evolution for time step dt transforms voltage $V_{n,i}$ into $V_{n+1,i}$, which means for the integer labels:

$$\{n, i\} \xrightarrow{dt} \{n + 1, i\} \tag{2.9}$$

Now, if initial state of neuron is given as floating point number $V \in [0; V_0[$, then one needs to find its approximation $V_{n,i} \in \mathbb{V}_{\alpha,N}$, as described above. The precision of this approximation is:

$$|V - V_{n,i}| < \Delta V_0/N \tag{2.10}$$

The required approximation value of n should satisfy the following relation: $\alpha^{n+1}V_0 \leq V < \alpha^n V_0$, which gives:

$$n = - \lceil \log_{\alpha}(V_0/V) \rceil - 1 \tag{2.11}$$

where $\lceil x \rceil$ denotes the integer part of x . Now, with n found, we determine the i value from the following relation: $\alpha^{n+1}V_0 + i c_n \leq V < \alpha^{n+1}V_0 + (i + 1)c_n$, which gives:

$$i = \lceil (V - \alpha^{n+1}V_0) / c_n \rceil \tag{2.12}$$

Now, evolution of LIF state can be expressed in integer numbers as follows. For initial value of voltage, $V(0) < V_0$, we find its integer representation/approximation $\{n, i\}$ in accordance with Eqs. (2.11), (2.12). The pure decay evolution then goes as displayed in (2.9). In order to describe influence of input impulse with magnitude h on the state $\{n, i\}$ we calculate the voltage $V_{n,i}$ in accordance to (2.8). The voltage after receiving input impulse becomes $V_{new} = V_{n,i} + h$. If $V_{new} \geq V_0$, then neuron produces an output impulse and ends in the state “empty” with $V = 0$. Otherwise, the new integer state, $\{n', i'\}$, can be found with (2.11), (2.12) applied to the V_{new} instead of V .

The procedures given above define the dynamical system intLIF in which a neuronal state is described with two integers, $\{n, i\}$, with additional unique state “empty” attained just after firing.

3. Results

3.1. Coding

Neuronal state is described by three integers: `int n,i; char empty;` If `empty == 1` then the neuron is in its resting state with $V = 0$. If `empty == 0` then V can be calculated from the `n,i` in accordance with (2.8). Eq. (2.8) gives the following C-code for calculating V from known integer state `\{n,i,empty\}`:

³ We do not consider the last bin from (2.6), because we do not expect the state V will ever fall into it, except for just after firing, when $V = 0$ exactly.

Table 1

Parameters of simulating algorithm.

Random number generators	MT19937, knuthran2002, taus113
Number of different seeds for each	10
Testing duration, real time	1 h
Max value for N	$N_{\max} = 10^9$
Min value for dt	$dt_{\min} = 0.001$ ms
Initial N	10
Initial dt	0.1, 0.01, 0.001 ms

Table 2

Physical parameters used in simulation.

Threshold depolarization, V_0	20 mV
Input impulse height, h	0.25, 0.5, 1., 2., 4., 8., 16. mV
Membrane relaxation time, τ	10., 20., 40. ms
Input stream intensity, λ	0.4, 0.8, 1.60, 3.20, 6.40 ms^{-1}

```
V = empty ? 0 : pow(al, n)*V0*(al + (double)i/N*(1 - al));
```

where V_0 stands for V_0 and al stands for α . After firing, `empty == 1`.

Eq. (2.11) gives the following C-code for calculating n :

```
n = - floor(log(V0/V)/log(al)) - 1;
```

Similarly for Eqs. (2.7), (2.12).

3.2. Testing

For testing the intLIF simulation paradigm, both intLIF and fpLIF models were stimulated with the same random stream of impulses. The stream with exponential distribution of inter-spike intervals (ISI) was generated with random number generator from the GNU Scientific Library.⁴ Generators of three types were used, each with a number of different seeds, see Table 1.

Each ISI obtained from the generator as floating point number was approximated with a value from \mathbb{T} by rounding to the nearest integer number of time steps dt :

```
double ISI = gsl_ran_exponential (r, mu);
int iISI = rint(ISI/dt);
ISI = dt*iISI;
```

where `gsl_ran_exponential (r, mu)` — is the generator, `r` — is the pointer to global generator, `mu` — is the mean inter-spike interval of the exponential distribution; $\mu = 1/\lambda$, where λ is the intensity of Poisson stream, which is obtained from the generator and used as input stimulating stream. This way obtained ISIs were used to apply input impulses to both the intLIF and fpLIF models within a single program. The program starts with initial values for N , dt given in Table 1. With each input impulse, it is checked whether both fpLIF and intLIF react the same way, namely, either both fire, or both not fire. If the reaction for some input impulse is not the same, a new value for N is chosen by multiplying the current value by 10, and simulation starts anew from the beginning. If the maximum value for N is reached (see Table 1) and fpLIF and intLIF firing moments still do not the same through the whole simulation time, next option is to divide dt by 10. A single run of the program is considered successful if during 1 h of real time all input impulses produced the same reaction in both intLIF and fpLIF.

Several sets of physical parameters, which cover a physiologically reasonable range, were used in the testing, see Table 2.

The testing was made for any combination of parameters from both Tables 1 and 2. Some characteristic numbers of input and output streams are given in Table 3.

⁴ See <http://www.gnu.org/software/gsl/>.

Table 3

Characteristics of streams of impulses during 1 h of real time testing.

Minimal number of input impulses	$\sim 1.4 \cdot 10^6$
Maximal number of input impulses	$\sim 23.4 \cdot 10^6$
Minimal number of output impulses	0
Maximal number of output impulses	$\sim 11.7 \cdot 10^6$

As a result of testing, it was found that for any combination of parameters it is possible to ensure that all firing moments of intLIF and fpLIF are identical by choosing proper N and dt values. The decisive factor, which determines whether all spiking moments of intLIF and fpLIF coincide, is the precision of approximating the interval of possible voltages, $[0; V_0]$, with discrete values from $\mathbb{V}_{\alpha, N}$ as compared to h . This relative error, δV , can be estimated based on (2.5) and (2.10): $\delta V = (1 - \alpha)V_0/(Nh)$, which for small $\frac{dt}{\tau}$ gives: $\delta V = (dt V_0)/(\tau N h)$. For testing, the biggest (worst) value of $\delta V = 0.08$. Actually, this is the initial value, which never gives a good approximation. In the process of running test, δV is improving up to $3.1 \cdot 10^{-14}$ by adjusting approximation parameters dt and N as described above. In the testing performed, it appeared that having $\delta V \leq 2.0 \cdot 10^{-11}$ guarantees that sequences of spiking moments of intLIF and fpLIF are identical. For a larger values of δV , the difference between the two sequences may appear, which is characterized by several misplaced spikes to several hundreds and more.

4. Discussion

The idea of this investigation was developed due to the following problem. Expect a reverberating neuronal network is running free after initial stimulation. In some cases, depending on the initial stimulus, the dynamic will stop with all neurons free of any excitation. In other, more interesting cases the dynamic will go indefinitely long, finally entraining onto a periodic regime. There may be a number of different periodic regimes, depending on the stimulus applied. In order to figure a periodic regime by means of computer simulation, one needs to be able to observe that a complete state of the network is exactly the same at two different instances of time. If the state is described by floating point numbers, the exact equality of states cannot be established. Some neuronal models can be naturally described in terms of integers, e.g. [11]. This is not the case for the leaky integrate-and-fire model due to exponential decay of excitation in it. For the LIF model, the equality of states can be established only approximately. This might not be a problem if a reverberating network has only a single attractor. But normally it has many, see [11,12]. In this case, even a small difference in the neuronal states the net is composed of may bring about different attractor states, or different states may be treated as identical bringing about not true periodic regime. Describing LIF with integers helps to avoid the problems inherent to the floating point arithmetic in computers, which become essential in the task described above.

In numerical simulation of a dynamical system adaptive algorithms are normally used, when the time step dt value is increased/decreased during calculations in order to make calculations faster and more precise. This works perfect if it is necessary to calculate the system's state at some future moment of time starting from some initial state. To determine periodic regimes in a reverberating network, it is instead necessary to calculate the whole trajectory during some interval of time. In this case, the straightforward way is to approximate that interval with equidistant discrete points as in (2.2) and calculate states of the system in those points. Therefore, paradigm of fixed time step is used here for a single neuron.

Description of neuronal state (voltage) with a pair of integers $\{n, i\}$ does not exempt the intLIF model from using machine floating point numbers. Indeed, in Eqs. (2.8), (2.11), (2.12), operations with floating point numbers are explicitly involved. Nevertheless, the pure decay evolution, as it is described in (2.9) goes without rounding errors. The underlying to (2.9) values from $\mathbb{V}_{\alpha, N}$ are always the same for the same $\{n, i\}$. With obtained input impulse, calculation of $V_{n,i} + h$ involves a rounding error. The error is immediately cleared while calculating new $\{n, i\}$ by means of (2.11), (2.12). This allows describing states of LIF neuron in terms of integers in a consistent manner. Namely, different state labels $\{n, i\}$ correspond to different voltages from $\mathbb{V}_{\alpha, N}$ and vice versa.

It is not surprising that quality of approximation of fpLIF with intLIF depends on the density of points $\alpha^n V_0$ within the interval (2.1): the smaller is $(1 - \alpha)V_0$ the denser are the points, the better is the approximation. The density can be increased by decreasing dt . This even might allow to avoid introducing the second order bins. But, with small dt , and α close to 1, the last interval in the representation (2.6) may appear too long, $\alpha^{\text{INT_MAX}} V_0$ being too large. In this case, the approach proposed will fail to describe pure decay dynamics for a long time enough, when $V(t)$ becomes smaller

than $\alpha^{\text{INT_MAX}} V_0$. The problem could be avoided if to migrate to multiple precision arithmetic library⁵ and chose a considerably bigger integer instead of INT_MAX in the representation (2.6).

We used here the simplest possible model for LIF neuron. It seems, that technique offered in nn. 2.2.1, 2.2.2, above, can be extended to be valid for more elaborated LIF models, like those described in [1,2,7,10].

5. Conclusions

The intLIF paradigm for numerical simulation of leaky integrate and fire neuron is proposed. In this paradigm, neuronal state is described by two integers, $\{n, i\}$. The membrane voltage of LIF neuron can be calculated from $\{n, i\}$, if required. The LIF state change due to both leakage and stimulating impulses is expressed exclusively in terms of changing integers $\{n, i\}$. The intLIF paradigm is compared with the standard fpLIF simulation paradigm, where membrane voltage is expressed as machine floating point number, by stimulating both models with the same random stream of input impulses and registering the spiking moments of both models. It is concluded that approximation parameters of intLIF can be chosen in such a way that spiking moments of both models are exactly the same if expressed as whole number of simulation time steps, dt . Description of LIF states by integers gives a consistent numerical model, suitable for situations where exact comparison of states is necessary.

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⁵ See <https://gmpilib.org/>.