

From chaos to clock in recurrent neural net. Case study

A. Vidybida^{a,*}, O. Shchur^{a,b}

^a Bogolyubov Institute for Theoretical Physics, Metrologichna str., 14-B, Kyiv 03143, Ukraine

^b Laboratory of Biological Computation, Institute of Experimental Medicine, 43 Szigonyi str., H-1083 Budapest, Hungary

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ABSTRACT

What is the reason for complex dynamical patterns registered from real biological neuronal networks? Noise and dynamical reconfiguring of a network (functional/dynamic connectome) were proposed as possible answers. In this case study, we report a complex dynamical pattern observed in a simple deterministic network of 25 excitatory neurons with fixed connectome. After a short initial stimulation, the network is engaged into a complex dynamics, which lasts for a long time. Eventually, with no external intervention, the dynamics comes to a periodic one with a short period. The long transient is positively checked for being chaotic. We conclude that the complex dynamics observed is the output of neural computation performed in the process of neuronal firings and spikes propagation.

1. Introduction

Chaotic dynamics in the brain have been observed for a long time, for example, in electroencephalography (EEG) during sleep (Babloyantz et al., 1985) and olfactory perception (Skarda and Freeman, 1987). Further investigations reported chaotic dynamical patterns at all levels of a brain down to single cells and their membrane conductances, see references in Korn and Faure (2003). Chaos is now recognized as a normal state of a living organism (Pool, 1989). Furthermore, in most cases, excessive rhythmic (non-chaotic) activity in the brain is considered as a pathology and should be corrected (see e.g. Schiff et al., 1994 where a possibility for correction is reported).

Several mechanisms of electrical, chemical and biological nature are able to shape dynamics in a biological neural net, see survey in Breakspear (2017). In this report, we describe a complex dynamics in a fully connected deterministic network of 25 leaky integrate-and-fire (LIF) excitatory neurons placed at lattice nodes, Fig. 1. Propagation delays are taken proportional to the interneuronal distances. The network is initially stimulated with a short sequence of 25 input impulses, each triggering one of the 25 neurons. The sequence of the triggering moments constitutes the stimulus specificity. After the initial stimulation, the network runs on its own, without external influence and with no plasticity. A stimulus has been found which triggers a prolonged

seemingly chaotic behavior of the network's state parameters, such as neuronal voltages or interspike intervals. This type of dynamics lasts several orders of magnitude longer than the longest interneuronal communication delay. After that, the dynamics becomes periodic with a short period. In order to analyze the transient observed, we apply several tests for complexity and chaos to it. These tests are as follows: 0-1 test by Gottwald and Melbourne; permutation entropy; spectral entropy; sensitive dependence on initial conditions. All tests support the idea that the initial transient is chaotic. This kind of activity looks like an example of the transient chaos, Tél (2015). Remarkably, none of the used tests was able to predict based on the initial chunk of the transient whether the dynamics will fade or settle on a periodic mode and how long could it take. These questions will be addressed in further work.

2. Methods

2.1. Neural network

The network is similar to that used for numerical simulations in our previous paper (Vidybida and Shchur, 2017). The only difference is the number of neurons (25 instead of 9, see Fig. 1). Similar to Vidybida and Shchur (2017), simulation is made with time step $dt = 0.1$ msec. Henceforth, we use the term "tick" both for duration of the time step, and for the process of advancing the network's state for one time step.

* Corresponding author.

E-mail addresses: vidybida@bitp.kiev.ua (A. Vidybida), olha.shchur@bitp.kiev.ua, shchur.olha@koki.hu (O. Shchur).

URL: <http://vidybida.kiev.ua/> (A. Vidybida).

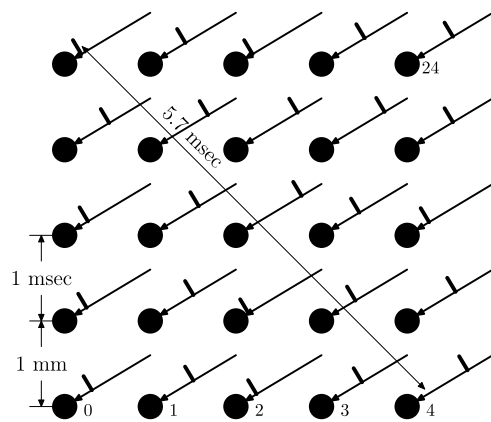


Fig. 1. Network used for simulations. All neurons are identical leaky integrate-and-fire excitatory neurons, with threshold voltage $V_{th} = 20$ mV, input impulse height $h = 0.999$ mV, membrane time constant $\tau_M = 20$ msec. The neurons are simulated with integer numbers as described in Vidybida (2019). Each of 25 bars indicate initial position in time of a triggering impulse from the stimulus.

2.2. Stimuli

Initial stimulation of the network is performed by applying a sequence of 25 identical triggering impulses, each for a corresponding neuron. The triggering impulse has magnitude greater than the firing threshold, thus able to trigger its target neuron immediately.¹ The stimulus used in this case study is displayed in the Figs. 1, 2. The stimulus can be depicted as usual as a spike-train, see Fig. 2 (right), but each triggering spike in the train is destined for its own neuron. Thus, due to initial stimulation, each neuron obtains single triggering input impulse at a specified moment. The set of all 25 triggering moments constitutes the stimulus specificity. After the stimulus is discharged entirely (it takes 4 msec for the stimulus used), the network's dynamics unfolds on its own. No further external intervention is involved and no plasticity or noise is considered.

2.3. Data acquisition

During free run after stimulation, the network state at any moment consists of 600 integers characterizing states of all 600 interneuronal connections, and 25×4 integers characterizing states of all 25 neurons. The state of a connection is represented by a single integer indicating after how many ticks the propagating spike will reach its target neuron. If a connection does not convey a spike, its state is marked as -1. In the state of a neuron, the first two integers represent depolarization with whole numbers. The other two report whether the neuron is in fire or empty state, see details in Vidybida (2019). In order to determine the moment when a periodic regime starts, we add the network state after each tick to a C++ container (actually, the hash of a state) until we meet the state, which is already in the container. The moment of the first appearance of that state is just the moment of entraining onto a periodic regime. After finding an interesting trajectory, we write it on a disc and analyze by several methods.

3. Results

3.1. Dynamics observed

If stimulated with stimulus displayed in Figs. 1, 2, the network demonstrates the following behavior. During long time (1569.4821 s = 26 min 9.5 s) the dynamics is seemingly chaotic. This can be seen from histograms of interspike intervals (ISIs) for a single neuron in different chunks of the transient, Fig. 3, and the sets of inter-spike intervals for

all 25 neurons, Fig. 4(left), and from the time course of the compound voltage V_{sum} , Fig. 5(left), where

$$V_{sum} = \sum_{i=0}^{24} v_i, \quad (1)$$

and v_i is the voltage in the neuron # i . Then, within a short time (less than 3 s), dynamics simplifies and eventually turns into periodic with period duration of 10.4 msec. In the periodic regime, 24 of 25 neurons fire with constant ISI duration 5.2 msec. One neuron fires ISIs 5.1 and 5.3 msec long alternately, e.g. Fig. 4(right). We consider the duration of the transient as relatively long. Indeed, time needed for a spike to cross the diagonal is 5.7 msec, Fig. 1. Thus, the diagonal crossing may happen $1569.4821 \text{ sec} / 5.7 \text{ msec} = 275348$ times before the dynamics settles down to periodic regime. Such a long transient appears as an independent self-reliant dynamics, which can be analyzed separately.

3.2. Analysis

In order to analyze the dynamics, we have chosen 10 chunks of the trajectory in the following way. Each chunk has the same duration. Chunks 1..10 follow one another and chunk # 9 ends just at the beginning of periodic regime. At this same moment chunk #10 starts, see Fig. 6. Three different durations of chunks have been considered, namely, 5200 ticks (50 periods) and 15600 ticks (150 periods), as well as 1743869 ticks (the first nine chunks cover the transient part of the trajectory entirely). As a parameter to analyze we take voltage at a single neuron, v_i , $i = 0, \dots, 24$, (several neurons have been tested) and the sum of all 25 voltages, V_{sum} , (1), which is the analog of the local field potential. Since results are similar for different parameters, we present them only for V_{sum} (see below).

3.2.1. 0-1 test for chaos

Here we use the binary test for chaos proposed by G. A. Gottwald and I. Melbourne, Gottwald and Melbourne (2009). In this test, a sequence of data

$$V = \{V_0, V_1, \dots, V_{N-1}\} \quad (2)$$

can be checked for being chaotic or regular. In our case $V_i = V_{sum}(i)$, where $i = 0, 1, 2, \dots, N-1$, is a tick number within a chunk and N is the chunk length. V is considered as being either regular or chaotic depending on the behavior of auxiliary two-dimensional trajectory

$$(p_c(n), q_c(n)), \quad n = 0, 1, \dots, N-1 \quad (3)$$

constructed from V as follows:

$$p_c(n) = \sum_{0 \leq i \leq n} V_i \cos(i \cdot c), \quad q_c(n) = \sum_{0 \leq i \leq n} V_i \sin(i \cdot c). \quad (4)$$

Here c is a real number from a set of 1000 equidistant values in the interval $(0, \pi)$. If the mean square displacement of $(p_c(n), q_c(n))$ asymptotically grows linearly with n , then V is considered chaotic, otherwise if $(p_c(n), q_c(n))$ stays in a bounded domain, then V is regular. The asymptotic mean square displacement is defined in Gottwald and Melbourne (2009, Eq. (2.1)) as follows:

$$M_c(n) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{0 \leq k < N} ((p_c(k+n) - p_c(k))^2 + (q_c(k+n) - q_c(k))^2), \quad (5)$$

which expects infinite number of points in V . In our case, we have a finite number of points from the end of stimulation to the entrainment onto the periodic regime. Moreover, we analyze dynamics in 9 consecutive chunks before the periodic regime starts, see Fig. 6. This limits possible value of N in (5) by 1743869. Thus, the modified for our case mean square displacement in each chunk is calculated as follows:

$$M_c(n) = \frac{1}{N_1} \sum_{0 \leq k < N_1} ((p_c(k+n) - p_c(k))^2 + (q_c(k+n) - q_c(k))^2), \quad (6)$$

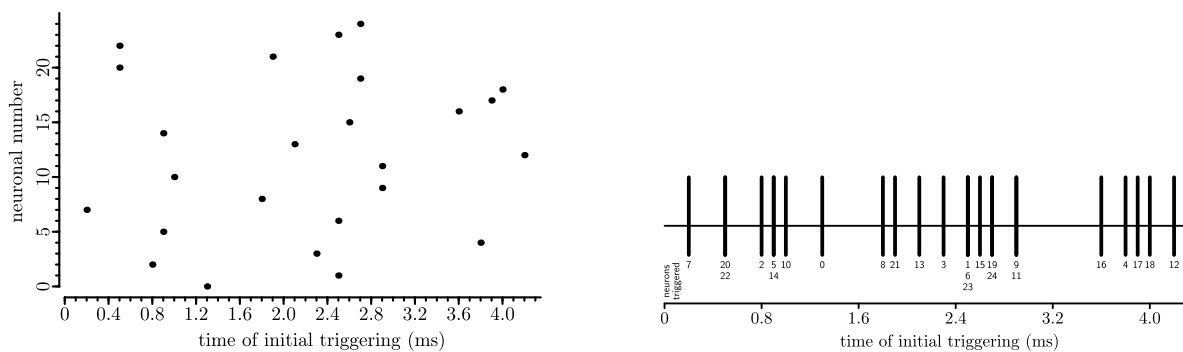


Fig. 2. Initial stimulus used. Left: X-coordinate of a dot indicates the moment at which input triggering spike enters a corresponding neuron; Y-coordinate is the corresponding neuron number. Right: the same stimulus displayed as a spike train. Numbers under a spike indicate neurons triggered by this spike.

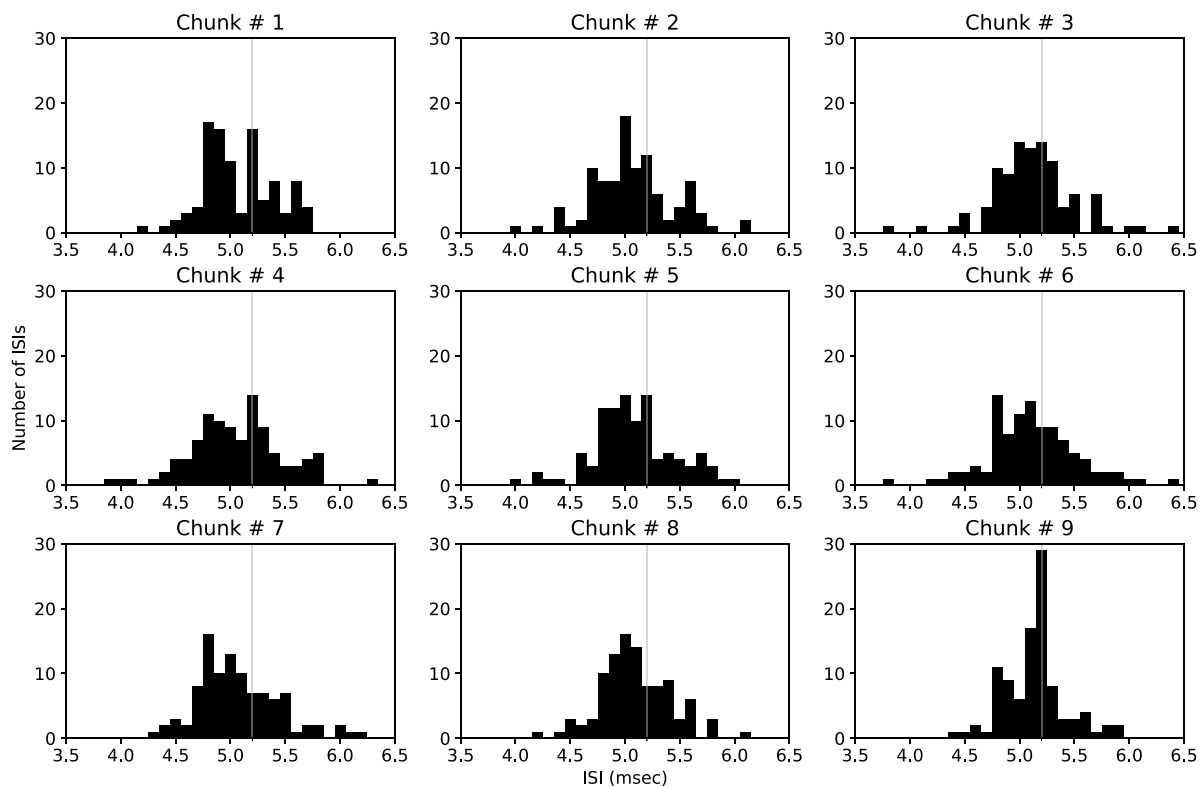


Fig. 3. The ISIs histograms for the chunks # 1-9 for the neuron # 24. Here, chunk duration is 0.52 s. Grid lines in all figures correspond to the ISI duration 5.2 msec. Note that, in the periodic regime, the neuron # 24 fires with constant ISI duration 5.2 msec.

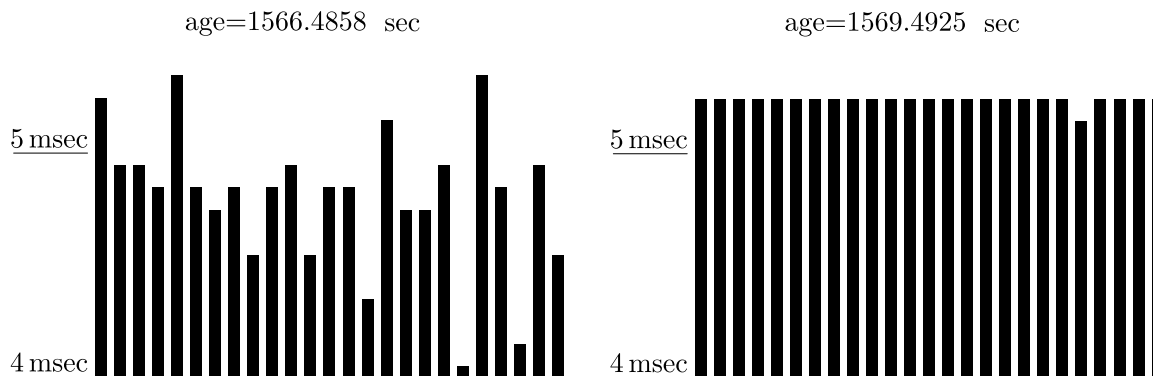


Fig. 4. Sets of most recent ISIs for all 25 neurons at time moments specified above as “age”. Typical ISI set in chaotic (left, 3 s to periodic regime) and periodic (right) regime.

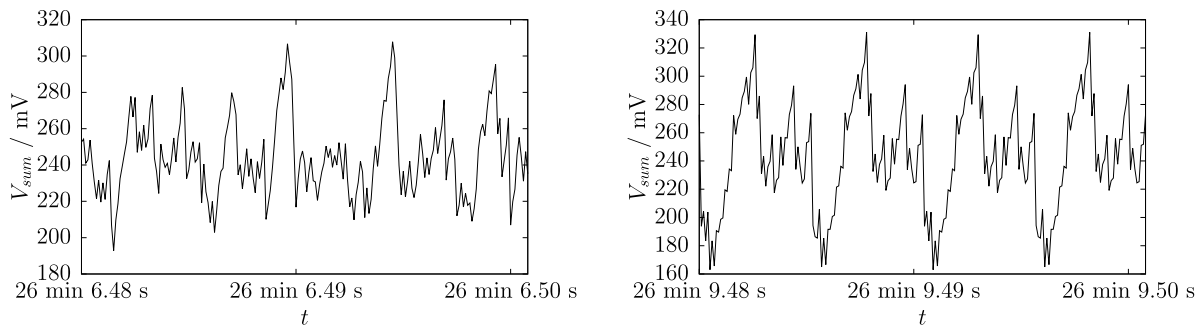


Fig. 5. The time course of the compound voltage after establishing the periodic regime (right, two periods long), and 3 s earlier (left).

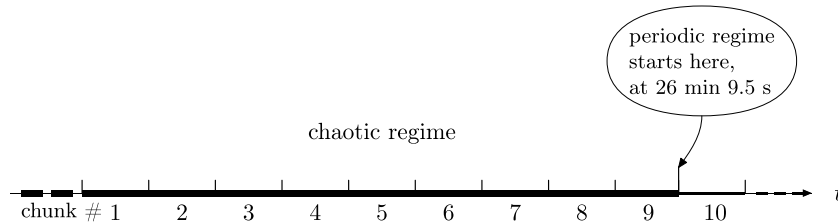


Fig. 6. Chunks used for calculations. Several chunk durations have been tested, see main text.

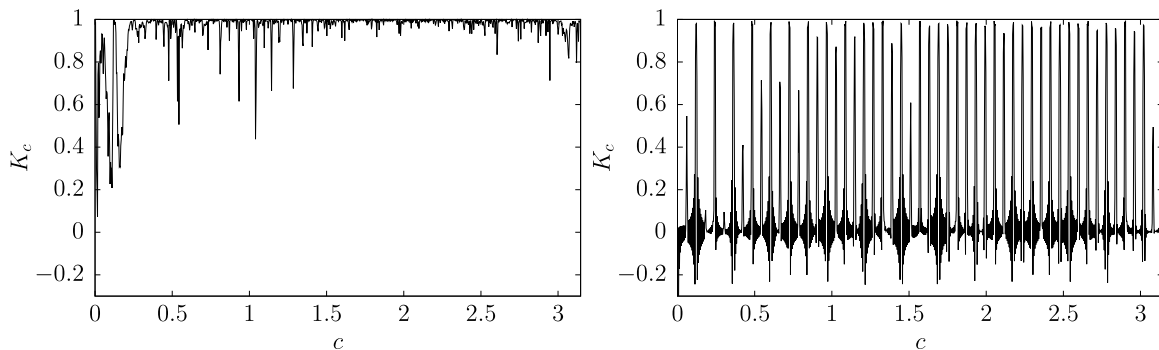


Fig. 7. Behavior of the correlation coefficient K_c as a function of c -value. Left: chunk #9, right: chunk #10 (periodic regime). Here, chunk duration is 0.52 s; 1000 equidistant values of $c \in [0; \pi]$ were used for calculations.

Table 1
Result of 0-1 test for chaos in each chunk. Here, chunk duration is 0.52 s.

Chunk number	1	2	3	4	5	6	7	8	9	10
Median of K_c	0.992	0.994	0.993	0.994	0.993	0.994	0.993	0.994	0.992	0.026

where $n = 0, 1, \dots, n_1 = 1000$, and the following condition is satisfied:

$$N_1 + n_1 \leq N,$$

where N is the chunk length. Mean square displacement $M_c(n)$ oscillates with n , which impairs convergence. The oscillations can be subtracted from $M_c(n)$ as it is proposed in Gottwald and Melbourne (2009, Eq. (2.3)). The resulting quantity $D_c(n)$ is checked for linear growth by correlation method, see Gottwald and Melbourne (2009, Sec. 3.2). The correlation coefficient K_c between sequences $\{1, 2, \dots, n_1\}$ and $\{D_c(1), D_c(2), \dots, D_c(n_1)\}$ is calculated for different values of c , and median in the obtained set K_c was found. Resulting median for the first nine chunks of the trajectory is close to 1 (see Fig. 7, Table 1) qualifying those chunks as chaotic. All three values for N mentioned above in Section 3.2 were tested with $n_1 = 1000$. The results are similar.

3.2.2. Permutation entropy

Complexity of trajectory $V_{sum}(t)$ in different chunks was also analyzed by calculating permutation entropy in each chunk. The permutation entropy method is proposed for estimating complexity of

trajectories of a dynamical system, see Bandt and Pompe (2002). In order to apply this method to a sequence of data (2), one needs to chose an embedding dimension $D > 1$ and create a sequence of embedding vectors $\mathbb{V} = \{\mathbf{V}_0, \mathbf{V}_1, \dots, \mathbf{V}_{N-D}\}$, $\mathbf{V}_i \in \mathbb{R}^D$, $i = 0, 1, 2, \dots, N - D$, where $\mathbf{V}_i = (V_i, V_{i+1}, V_{i+2}, \dots, V_{i+D-1})$. An additional parameter of the embedding procedure is delay, which we choose 1 here. Further step in the method is to find for each \mathbf{V}_i a permutation π_i , which arranges its components in ascending order. π_i is called the order pattern of \mathbf{V}_i . Having a sequence of order patterns $\Pi = \{\pi_0, \pi_1, \dots, \pi_{N-D}\}$, we calculate the probability p_i of any of $D!$ possible patterns by dividing the number of its occurrences in Π by the total number of elements in Π . The permutation entropy of \mathbb{V} is the Shannon entropy of the probability distribution $p(\pi_i)$:

$$S(\mathbb{V}) \equiv S(\Pi) = - \sum_{i=0}^{M-1} p(\pi_i) \log(p(\pi_i)),$$

where M is the number of different permutations in the Π . In this work we use a modification of this method, namely arithmetic entropy,

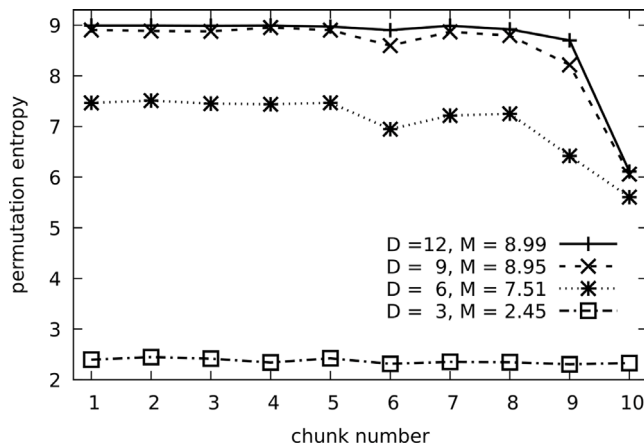


Fig. 8. Permutation entropy for different embedding dimensions D calculated for V_{sum} in chunks shown in Fig. 6. M is the maximal value of the permutation entropy for each D . Here, chunk duration is 0.52 s. The embedding delay $\tau = 1$.

which is exempt of combinatorics, see Vidybida (2020). Both methods deliver the same value for entropy provided that in any embedding vector all D components are different, see Vidybida (2020, Theorem A.1). In our simulation, we registered voltages with nine decimal places, therefore equalities are improbable. Additionally, we checked for equality between components of embedding vector and observed none in V_i . The resulting entropy values are shown in Fig. 8. From these data we see that the trajectory $V_{sum}(t)$ has high complexity, roughly the same for the first 8 chunks. In the ninth chunk, complexity decreases slightly, and falls to lower values at the tenth chunk. In the periodic regime, the permutation entropy is still considerably high. This can be understood having in mind that the periodic regime itself is still quite complex, see Fig. 5 (right). For embedding dimensions considered, except of $D = 3$, periodic part of the trajectory produces enough different order patterns.

3.2.3. Spectral entropy

Spectral entropy is one of nonlinear dynamics and chaos theory methods used to analyze EEG signals, Rodríguez-Bermúdez and García-Laencina (2015). Different entropy measures can be used to detect epileptic seizure on EEG. In a review (Rajendra Acharya et al., 2015), it was concluded that the spectral entropy is among the best entropy measures to perform in this task. Also different entropy measures, including the spectral entropy, are used to classify emotions from EEG in a brain-computer interface, see recent review (Patel et al., 2021). Additionally, applied to local field potential measurements, correlation of time varying spectral entropies is used to detect synchrony in neural networks (Kapucu et al., 2016). Therefore, we have tested the transient with this method.

Firstly, different chunks of the trajectory $V_{sum}(t)$ were analyzed by calculating the spectral power density (PSD) and subsequently the power spectral entropy (PSE). The algorithm used to calculate PSD is as follows. In the current work, during the course of computer simulation, the sum of voltages of all 25 neurons V_{sum} (1) was sampled at discrete times with step dt . For one chunk, sampled V_{sum} is given by V as in (2) with $N = 5200$, where N is the size of a chunk. Firstly, one needs to calculate the discrete Fourier transform of the sequence V :

$$\tilde{V} = \{\tilde{V}_0, \tilde{V}_1, \dots, \tilde{V}_{N-1}\}. \quad (7)$$

Then, from the discrete Fourier transform of V , the PSD at the frequency $f_k = \frac{k}{Ndt}$, $k = 1, \dots, N/2$, can be calculated using the following expression:

$$PSD(f_k) = \frac{|\tilde{V}_k|^2 dt}{N}. \quad (8)$$

The PSD was calculated for all ten chunks. For the chunks #1 and #10, the PSD is depicted on Fig. 9. Note that on the chunk #10 (periodic activity, lower panel of Fig. 9) the frequency 1/5.2 kHz and its harmonics have the most power, while on the chunk #1 (upper panel of Fig. 9) wide peaks in the vicinity of the frequency 1/5.2 kHz and its harmonics are present.

The PSE, or the Shannon spectral entropy, is an application of Shannon entropy expression to the power spectral density components (Rajendra Acharya et al., 2015). To calculate the PSE, the PSD is usually normalized with the total power:

$$PSD_{norm}(f_k) = \frac{PSD(f_k)}{\sum_{l=1}^{N/2} PSD(f_l)}. \quad (9)$$

Then the power spectral entropy is given by the following formula:

$$PSE = -\frac{1}{\log_2(N/2)} \sum_{k=1}^{N/2} PSD_{norm}(f_k) \log_2(PSD_{norm}(f_k)). \quad (10)$$

Note that here the PSE is normalized with $\log_2(N/2)$, which is the maximal PSE of a white noise having equal intensity at all frequencies.

The normalized PSE was calculated for chunks #1-10. The results of calculations are depicted on Fig. 10. The PSE is roughly the same for the chunks #1-9 during the relaxation to periodic activity, and sharply drops on the chunk #10 for periodic activity.

3.2.4. Sensitivity to small perturbations

It is known that for chaotic dynamical systems small perturbations of initial state are able to produce a large divergence of resulting trajectories. In our case, initial state is achieved at the end of a stimulus applied to the standard state with empty neurons and axons. Naturally, a small difference between initial stimuli results in a small difference in the network's initial state. Therefore, and having in mind that a network is aimed at processing initial input into an output, we consider here small perturbations of the initial stimulus. A stimulus specificity is determined by moments of initial triggering of each neuron, see Fig. 2. Since we have a finite time step $dt = 100 \mu s$, the smallest possible perturbation of the stimulus can be prepared by shifting triggering moment of a single neuron by $\pm dt$. This gives 50 different stimuli/cases characterized by the smallest possible deviation from the initial one. Each perturbed stimulus was applied and resulting dynamics was analyzed. The analysis summary is presented in the Table 2, see also the Table 3, where simulation is made for increased synaptic strength. We see that some of the perturbed stimuli cause dynamics which ends up with periodic regime with other period duration than the unperturbed one. This can be treated as sensitive dependence on initial conditions/stimulus, which characterizes chaotic dynamics. In this study, we did not compare terminal (periodic) dynamics within the pool of those with the same period duration. Previously, Vidybida and Shchur (2017), we observed several types of qualitatively different dynamics with the same period. This might be an additional argument in favor of sensitive dependence on the initial conditions.

4. Conclusions and discussion

In this report, in a simple deterministic neural network simulated on a PC, we have observed a remarkable dynamics in which, after a short initial stimulation, a chaotic transient comes to a periodic activity. We have checked the transient observed for being chaotic by several known methods. In this connection it should be noted that standard definition of chaos is made for systems with continuous trajectories in a metric space (see a discussion of the exact Devaney's definition of chaos in Banks et al., 1992). In our case, trajectories are not continuous: each neuronal firing produces a discontinuity. Also, due to the numerical modeling, our trajectories are confined in a finite set of state points. Therefore, straightforward application of the chaos definition made for

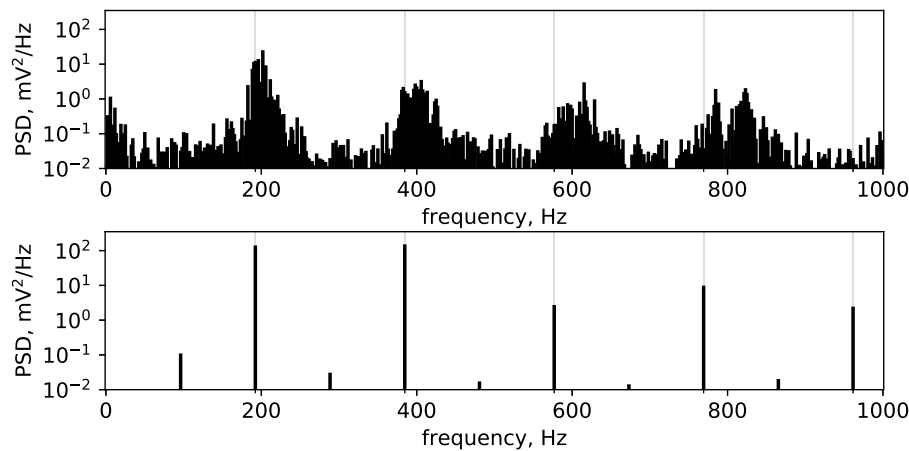


Fig. 9. The power spectral density (PSD) of the sum of voltages of all 25 neurons $V_{sum}(1)$ calculated on the chunks # 1 (upper panel) and #10 (lower panel). Only frequencies up to 1 kHz are shown. Grid lines in both figures correspond to the frequency 1/5.2 kHz and its harmonics.

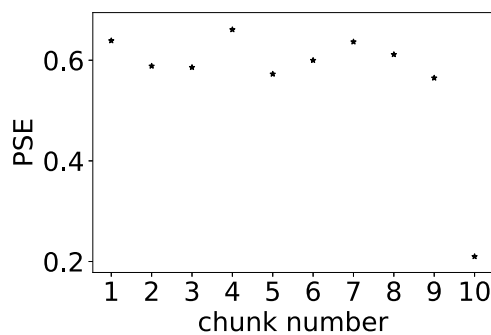


Fig. 10. The normalized power spectral entropy (PSE). X-coordinate of a star indicates the chunk number.

Table 2

Results of checking stability with respect to the perturbations of the initial stimulus. All 50 cases of minimally perturbed initial stimulus plus the initial stimulus itself are presented. The column #3 indicates the number of perturbed stimuli resulting in periodic dynamics with the period duration specified in the column #1. The initial (unperturbed) stimulus case is added in the corresponding row (period duration is 10.4 ms).

Period, ms	Number of spikes each neuron emits per period	Number of cases	Relaxation time, s
0 (activity fades)	0	2	0.08, 0.09
9.6	2	3	60 – 436
9.7	2	2	30, 257
9.8	2	2	344
10.0	2	2	368
10.4	2	35	26 – 1569 (26 min)
41.6	8	5	36 – 722

Table 3

Terminal regimes found for the same set of stimuli as in the Table 2, but in the network with the synaptic strength increased from $h = 0.999$ mV to $h = 1.0$ mV. The initial stimulus ends up in this case with period duration 41.6 ms instead of 10.4 ms. The relaxation times here are between 0.045 min and 9.85 min.

Period, ms	6.3	9.6	9.8	10.0	10.4	28.8	29.0	41.6
Number of cases	1	9	2	3	5	2	1	28

continuous systems does not make sense in our case. The methods applied here (0-1 test for chaos, spectral and permutation entropy estimates of complexity) do not expect continuity and can be applied to discontinuous trajectories. The appealing property of chaos known as

sensitive dependence on initial conditions expects a possibility to consider infinitesimally small perturbations of initial conditions, see Banks et al. (1992). In our case, the smallest possible perturbations are finite. Some of these perturbations result in trajectories differing qualitatively from the unperturbed one, see Tables 2 and 3.

In a real biological network, a regime ending with a periodic activity would be a real disaster (maybe, except of a central pattern generator). A real biological network has various mechanisms for interrupting perfect periodicity. Among those mechanisms, the stochastic nature of activity will break periodicity, as it could also be supposed by comparing either rows in the Table 2, or columns in the Table 3, where sensitivity to small perturbations is demonstrated. Also plasticity, like a change in the synaptic strength, could result in the change of a terminal regime. This can be seen in our specific case by comparing the whole Table 2 with the Table 3. Also, real biological network obtains either internal or external stimulation, or both all the time. Furthermore, the stimulation can be also chemical. Taking additionally into account that we consider the fully connected network, it would be difficult, if not impossible, to link the dynamics studied to a concrete observation made on a living object.

Our aim was to uncover the neural computation that can be performed in such a simplified, stripped of some essential biological features network. A natural question arises: What is meant under the neural computation in the context of this paper? In order to propose something grounded in the real world, like “the fruit fly computes its path to the apple”, it would be necessary to endow our randomly generated stimuli with some real meaning, and to interpret the network’s activity in terms of a specific purpose. This was not a goal of this study. There are several views on possible mechanisms of the neural computation, see Piccinini and Bahar (2013). We share the one expressed in Piccinini and Bahar (2013) that “neural computation is *sui generis*”. Correspondingly, as the neural computation in our particular case we mean the transformation of spatio-temporal pattern of activity in the network due to the neurons receiving impulses, firing and sending them out, see also Izhikevich (2006). Observation of very long transients, which look as being chaotic, shows that an extremely simplified recurrent neural network is able to perform rather complicated computations.

Does this kind of behavior is typical for a network of this type? Firstly, it should be mentioned that the states space of the network studied is really big: it is composed as the product of state spaces of all 25 neurons and 600 axons. Therefore, the network’s trajectory has enough possibilities to go through the space without reproducing one of its previous states. This may take a long time. But, as it can be seen from the Table 2, it is not necessary to exhaust all possible states in order to reproduce one previously passed state for the second time

indicating that the dynamics is already in a periodic regime. Usually, a stimulus sends the network to a terminal regime much faster. Our empirical observations indicate that, in the considered network, 10 to 15 randomly generated stimuli applied to the network, starting each time from its initial state, are enough to find a mode with a transient around 10 min long.

This is not the first observation of this type in a simulated neuronal network, see, e.g. [Zumdieck et al. \(2004\)](#), [Zillmer et al. \(2009\)](#). In [Zumdieck et al. \(2004\)](#), networks of phase oscillators are studied. The nature of a phase oscillator neuronal model is such that it produces output spikes even without external stimulation. This allowed not to use external stimuli at all. The authors observed long transients depending on the initial state chosen in diluted networks of up to 100 neurons with identical interneuronal transmission delays. It is impossible to compare the duration of those transients with ours because the time unit is not specified in [Zumdieck et al. \(2004\)](#). Also, long chaotic transients were absent in [Zumdieck et al. \(2004\)](#) for globally coupled case considered here. In [Zillmer et al. \(2009\)](#), a network of up to 10 000 inhibitory neurons is considered. Due to the absence of excitatory neurons, a permanent drive is required for the dynamics not to fade. The drive is chosen as the same constant excitatory current applied to each neuron. The interneuronal transmission delays are all taken to be 2 ms. Depending on initial conditions, chaotic transients of up to 10 min long were observed. In both studies, terminal state is characterized by a high degree of synchronicity between individual neurons. This is not the case for our network: in the periodic regime, neurons fire at different moments, but with the same mean frequency. Our simulation protocol is to apply a stimulus and to follow the network's dynamics. The absence of stimuli in [Zumdieck et al. \(2004\)](#), [Zillmer et al. \(2009\)](#) does not make a big difference with our case: initial states used in [Zumdieck et al. \(2004\)](#), [Zillmer et al. \(2009\)](#) can be thought as prepared by some initial stimuli remaining behind the scene. In our case, the initial state is always the same — the network is free of any activity.

Reasons for a neural network to reproduce chaos could be various, see [Faure and Korn \(2001\)](#) for discussion. Our modeling algorithm operates in whole numbers, thus excluding a possibility of rounding errors. Also, no noise is considered. It seems that the only reason for complex behavior in our network is the neural computation performed due to neuronal firings and resulting in rearrangement of interspike intervals in accordance with the rules imposed by the interneuronal communication delays and the LIF neuronal model parameters. This is a self-organization in the time domain envisioned by D. M. MacKay, ([MacKay, 1962](#)). Our finding is that in a network, the process of self-organization can be quite long, passing through complex states and essentially dependent on the initial stimulus. Also, in a network, self-organization happens not only in the time, but also in the space domain. This is because it was observed that in some cases terminal regimes with the same period duration may have different spatial patterns of activity: neurons fire in different order and with different intervals between them. In our previous work ([Vidybida, 2011](#); [Vidybida and Shchur, 2017](#)) made for smaller networks similar behavior has been observed, but with shorter transients.

In this connection a question arises: To what extent does the structure of a neuronal network (“connectome”, [Sporns et al., 2005](#)) determine its function? From the point of view of physics the answer should be the following: completely. But it has appeared quite difficult to map brain structure to its function (if function is considered as a concrete dynamics evoked by a concrete stimulus) based exclusively on the connectome. The idea of a “dynamome” was proposed as some additional rules governing the dynamics, ([Bargmann and Marder, 2013](#); [Kopell et al., 2014](#)). In parallel, concepts of “functional connectome”, ([Biswal et al., 2010](#)), or “dynamic connectome”, ([Quiroga, 2020](#)), were proposed. In these concepts, connections in the brain can be functionally/dynamically reconfigured depending on the cognitive task, or neuromodulator presence. As a result, complex brain dynamics are generated. The case reported here, see also [Vidybida \(2011\)](#), [Vidybida](#)

and [Shchur \(2017\)](#), demonstrates that complex, stimulus dependent dynamic repertoires can also be generated without external intervention in a deterministic recurrent network with fixed connectome exclusively due to the numbers game performed in the process of neural computation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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