Simulation of the membrane voltage evolution in the odor presence

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Abstract

Here we describe the numerical algorithm used in the program for calculating ORN's response to odor stimulation.

1 Model of ORN

As model ORN we use the leaky integrate-and-fire model with fluctuating conductance input similar to that used in [1]:

$$c_m \frac{dV(t)}{dt} = -g_l(V(t) - V_{rest}) - n(t) g_{OR}(V(t) - V_e), \tag{1}$$

where

$V(t)$ — is the membrane voltage (denoted in the program either as `V1` or `V2`);

V_{rest} — is the resting voltage (denoted in the program as `Vrest` and read from the file `DATA.ORN`);

c_m — is the total capacity of ORN's membrane (denoted in the program as `cm`);

g_l — is the total leakage through it (denoted in the program as `gl`);

V_e — is the reversal potential for current through open OR channel (denoted in the program as `Vexcit` and read from the file `DATA.ORN`);

$n(t)$ — is the fluctuating number of bound ORs at moment t (denoted in the program either as `n1` or `n2`);

g_{OR} — is the conductance of single open OR channel (denoted in the program as `ge` and read from the file `DATA.OR`).

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The model presented in the Eq. (1) is, as usual, extended with the threshold voltage, V_{th} (denoted in the program as `Vth` and read from the file `DATA.ORN`): if $V(t)$ exceeds V_{th}, then the ORN fires a spike and $V(t)$ is reset to V_{rest}.

Our purpose is to run evolution as described above for some period T (denoted in the program as `T`) and to calculate the total number of spikes emitted (denoted in the program either as `fire1` or `fire2` for the odor O1 or O2, respectively).

1.1 Numerical algorithm

The total number of ORs bound with odor, $n(t)$ in (1) is a fluctuating quantity simulated with the help of random number generator as described in the document `stoch_proc_simulation.pdf` in this directory. Each new value of $n(t)$ is supplied at the beginning of each time step dt and is considered constant during the dt. Thus, transition from $V_k \equiv V(k\,dt)$ to $V_{k+1} \equiv V((k+1)\,dt)$ is governed by the following linear differential equation with constant coefficients:

$$\frac{dV(t)}{dt} = -\alpha_k V(t) + \beta_k, \tag{2}$$

with initial condition $V(k\,dt) = V_k$, where

$$\alpha_k = \frac{g_l + g_{OR}\, n_k}{c_m}, \quad \beta_k = \frac{g_l\, V_{rest} + g_{OR}\; n_k\, V_e}{c_m}, \quad n_k \equiv n(k\,dt). \tag{3}$$

Eq. (2) can be easily solved during interval $[k\,dt; (k+1)\,dt]$, which gives for the time dependence there:

$$V(k\,dt + t) = V_k e^{-\alpha_k\, t} + \frac{\beta_k}{\alpha_k}\left(1 - e^{-\alpha_k\, t}\right), \quad 0 \le t \le dt, \tag{4}$$

and for the next value of V:

$$V_{k+1} = V_k e^{-\alpha_k\, dt} + \frac{\beta_k}{\alpha_k}\left(1 - e^{-\alpha_k\, dt}\right). \tag{5}$$

We decide that there was a spike within $[k\,dt; (k+1)\,dt]$ if $V_{k+1} > V_{th}$, and there wasn't if $V_{k+1} \le V_{th}$. This is justifiable since the dependence on t in (4) is monotonous. In the case of spike, we replace V_{k+1} with V_{rest} and add 1 to the total number of spikes, `fire1` or `fire2`, before running the next time step, see files `run_trajec1.cpp`, `run_trajec2.cpp`.

The final values of `fire1` and `fire2` are used to calculate the ORN's selectivity, see file `logics_of_the_program.pdf` in this directory.

References

1 Alexandre Kuhn, Ad Aertsen, and Stefan Rotter. Neuronal integration of synaptic input in the fluctuation-driven regime. *The Journal of Neuroscience*, 24(10):2345, 2004.